CHAPTER 10 Introduction to Binomial Trees

A useful and very popular technique for pricing a stock option involves constructing a *binomial tree*. This is a diagram that represents different possible paths that might be followed by the stock price over the life of the option. In this chapter we will take a first look at binomial trees and their relationship to an important principle known as risk-neutral valuation. The general approach adopted here is similar to that in an important paper published by Cox, Ross, and Rubinstein in 1979.

The material in this chapter is intended to be introductory. More details on the use of numerical procedures involving binomial trees are in Chapter 17.

10.1 A ONE-STEP BINOMIAL MODEL

We start by considering a very simple situation: A stock price is currently \$20, and it is known that at the end of three months it will be either \$22 or \$18. We are interested in valuing a European call option to buy the stock for \$21 in three months. This option will have one of two values at the end of the three months. If the stock price turns out to be \$22, the value of the option will be \$1; if the stock price turns out to be \$18, the value of the option will be zero. The situation is illustrated in Figure 10.1.

It turns out that a relatively simple argument can be used to price the option in this example. The only assumption needed is that no arbitrage opportunities exist. We set up a portfolio of the stock and the option in such a way that there is no uncertainty about the value of the portfolio at the end of the three months. We then argue that, because the portfolio has no risk, the return it earns must equal the risk-free interest rate. This enables us to work out the cost of setting up the portfolio and therefore the option's price. Because there are two securities (the stock and the stock option) and only two possible outcomes, it is always possible to set up the riskless portfolio.

Consider a portfolio consisting of a long position in Δ shares of the stock and a short position in one call option. We calculate the value of Δ that makes the portfolio riskless. If the stock price moves up from \$20 to \$22, the value of the shares is 22Δ and the value of the option is 1, so that the total value of the portfolio is $22\Delta - 1$. If the stock price moves down from \$20 to \$18, the value of the shares is 18Δ and the value of the option is zero, so that the total value of the portfolio is 18Δ. The portfolio is

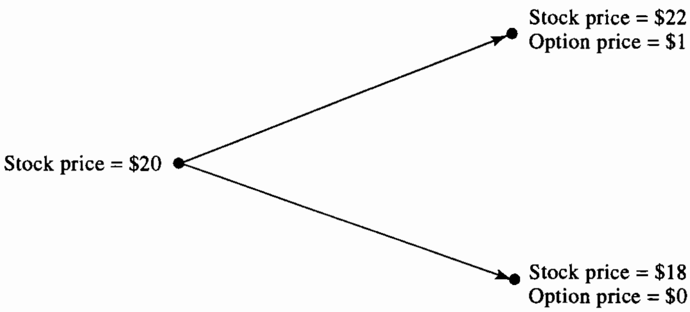

Figure 10.1 Stock price movements in numerical example

riskless if the value of Δ is chosen so that the final value of the portfolio is the same for both alternatives. This means

$$22\Delta - 1 = 18\Delta$$

or

$$\Delta = 0.25$$

A riskless portfolio is therefore

Long: 0.25 shares
Short: 1 option

If the stock price moves up to $22, the value of the portfolio is

$$22 \times 0.25 - 1 = 4.5$$

If the stock price moves down to $18, the value of the portfolio is

$$18 \times 0.25 = 4.5$$

Regardless of whether the stock price moves up or down, the value of the portfolio is always 4.5 at the end of the life of the option.

Riskless portfolios must, in the absence of arbitrage opportunities, earn the risk-free rate of interest. Suppose that in this case the risk-free rate is 12% per annum. It follows that the value of the portfolio today must be the present value of 4.5, or

$$4.5e^{-0.12\times 3/12} = 4.367$$

The value of the stock price today is known to be $20. Suppose the option price is denoted by f. The value of the portfolio today is

$$20 \times 0.25 - f = 5 - f$$

It follows that

$$5 - f = 4.367$$

or

$$f = 0.633$$

This shows that, in the absence of arbitrage opportunities, the current value of the option must be 0.633. If the value of the option were more than 0.633, the portfolio would cost less than 4.367 to set up and would earn more than the risk-free rate. If the value of the option were less than 0.633, shorting the portfolio would provide a way of borrowing money at less than the risk-free rate.

A Generalization

We can generalize the argument just presented by considering a stock whose price is S_0 and an option on the stock whose current price is f. We suppose that the option lasts for time T and that during the life of the option the stock price can either move up from S_0 to a new level, S_0u, or down from S_0 to a new level, S_0d ($u > 1$; $d < 1$). The proportional increase in the stock price when there is an up movement is $u - 1$; the proportional decrease when there is a down movement is $1 - d$. If the stock price moves up to S_0u, we suppose that the payoff from the option is f_u; if the stock price moves down to S_0d, we suppose the payoff from the option is f_d. The situation is illustrated in Figure 10.2.

As before, we imagine a portfolio consisting of a long position in Δ shares and a short position in one option. We calculate the value of Δ that makes the portfolio riskless. If there is an up movement in the stock price, the value of the portfolio at the end of the life of the option is

$$S_0u\Delta - f_u$$

If there is a down movement in the stock price, the value becomes

$$S_0d\Delta - f_d$$

The two are equal when

$$S_0u\Delta - f_u = S_0d\Delta - f_d$$

or

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d} \tag{10.1}$$

In this case, the portfolio is riskless and must earn the risk-free interest rate. Equation (10.1) shows that Δ is the ratio of the change in the option price to the change in the stock price as we move between the nodes.

If we denote the risk-free interest rate by r, the present value of the portfolio is

$$(S_0u\Delta - f_u)e^{-rT}$$

The cost of setting up the portfolio is

$$S_0\Delta - f$$

It follows that

$$S_0\Delta - f = (S_0u\Delta - f_u)e^{-rT}$$

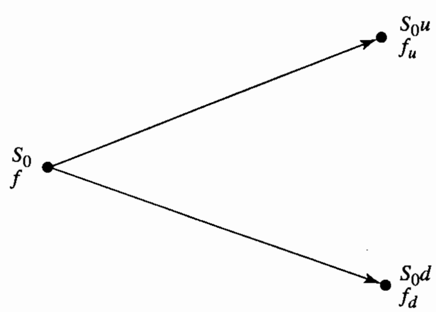

Figure 10.2 Stock and option prices in a general one-step tree

or

$$f = S_0\Delta - (S_0 u\Delta - f_u)e^{-rT}$$

Substituting from equation (10.1) for Δ and simplifying, we can reduce this equation to

$$f = e^{-rT}[pf_u + (1-p)f_d] \tag{10.2}$$

where

$$p = \frac{e^{rT} - d}{u - d} \tag{10.3}$$

Equations (10.2) and (10.3) enable an option to be priced using a one-step binomial model.

In the numerical example considered previously (see Figure 10.1), $u = 1.1$, $d = 0.9$, $r = 0.12$, $T = 0.25$, $f_u = 1$, and $f_d = 0$. From equation (10.3)

$$p = \frac{e^{0.12\times 3/12} - 0.9}{1.1 - 0.9} = 0.6523$$

and from equation (10.2)

$$f = e^{-0.12\times 0.25}(0.6523 \times 1 + 0.3477 \times 0) = 0.633$$

The result agrees with the answer obtained earlier in this section.

Irrelevance of the Stock's Expected Return

The option pricing formula in equation (10.2) does not involve the probabilities of the stock price moving up or down. For example, we get the same option price when the probability of an upward movement is 0.5 as we do when it is 0.9. This is surprising and seems counterintuitive. It is natural to assume that as the probability of an upward movement in the stock price increases, the value of a call option on the stock increases and the value of a put option on the stock decreases. This is not the case.

The key reason is that we are not valuing the option in absolute terms. We are calculating its value in terms of the price of the underlying stock. The probabilities of future up or down movements are already incorporated into the price of the stock. It turns out that we do not need to take them into account again when valuing the option in terms of the stock price.

10.2 RISK-NEUTRAL VALUATION

Although we do not need to make any assumptions about the probabilities of up and down movements in order to derive equation (10.2), it is natural to interpret the variable p in equation (10.2) as the probability of an up movement in the stock price. The variable $1 - p$ is then the probability of a down movement, and the expression

$$pf_u + (1-p)f_d$$

is the expected payoff from the option. With this interpretation of p, equation (10.2) then states that the value of the option today is its expected future value discounted at the risk-free rate.

We now investigate the expected return from the stock when the probability of an up movement is assumed to be p. The expected stock price at time T, $E(S_T)$, is given by

$$E(S_T) = pS_0u + (1-p)S_0d$$

or

$$E(S_T) = pS_0(u-d) + S_0d$$

Substituting from equation (10.3) for p, we obtain

$$E(S_T) = S_0e^{rT} \tag{10.4}$$

showing that the stock price grows on average at the risk-free rate. Setting the probability of the up movement equal to p is therefore equivalent to assuming that the return on the stock equals the risk-free rate.

In a *risk-neutral world* all individuals are indifferent to risk. In such a world investors require no compensation for risk, and the expected return on all securities is the risk-free interest rate. Equation (10.4) shows that we are assuming a risk-neutral world when we set the probability of an up movement to p. Equation (10.2) shows that the value of the option is its expected payoff in a risk-neutral world discounted at the risk-free rate.

This result is an example of an important general principle in option pricing known as *risk-neutral valuation*. The principle states that we can with complete impunity assume the world is risk neutral when pricing options. The resulting prices are correct not just in a risk-neutral world, but in other worlds as well.

The One-Step Binomial Example Revisited

We now return to the example in Figure 10.1 and illustrate that risk-neutral valuation gives the same answer as no-arbitrage arguments. In Figure 10.1, the stock price is currently \$20 and will move either up to \$22 or down to \$18 at the end of three months. The option considered is a European call option with a strike price of \$21 and an expiration date in three months. The risk-free interest rate is 12% per annum.

We define p as the probability of an upward movement in the stock price in a risk-neutral world. We can calculate p from equation (10.3). Alternatively, we can argue that the expected return on the stock in a risk-neutral world must be the risk-free rate of 12%. This means that p must satisfy

$$22p + 18(1-p) = 20e^{0.12\times 3/12}$$

or

$$4p = 20e^{0.12\times 3/12} - 18$$

That is, p must be 0.6523.

At the end of the three months, the call option has a 0.6523 probability of being worth 1 and a 0.3477 probability of being worth zero. Its expected value is therefore

$$0.6523 \times 1 + 0.3477 \times 0 = 0.6523$$

In a risk-neutral world this should be discounted at the risk-free rate. The value of the option today is therefore

$$0.6523e^{-0.12\times 3/12}$$

or \$0.633. This is the same as the value obtained earlier, demonstrating that no-arbitrage arguments and risk-neutral valuation give the same answer.

Real World vs. Risk-Neutral World

It should be emphasized that p is the probability of a up movement in a risk-neutral world. In general this is not the same as the probability of an up movement in the real world. In our example $p = 0.6523$. When the probability of an up movement is 0.6523, the expected return on the stock is the risk-free rate of 12%. Suppose that in the real world the expected return on the stock is 16%, and q is the probability of an up movement in the real world. It follows that

$$22q + 18(1-q) = 20e^{0.16\times 3/12}$$

so that $q = 0.7041$.

The expected payoff from the option in the real world is then

$$q \times 1 + (1-q) \times 0$$

This is 0.7041. Unfortunately it is not easy to know the correct discount rate to apply to the expected payoff in the real world. A position in a call option is riskier than a position in the stock. As a result the discount rate to be applied to the payoff from a call option is greater than 16%. Without knowing the option's value, we do not know how much greater than 16% it should be.[1] Using risk-neutral valuation is convenient because we know that in a risk-neutral world the expected return on all assets (and therefore the discount rate to use for all expected payoffs) is the risk-free rate.

10.3 TWO-STEP BINOMIAL TREES

We can extend the analysis to a two-step binomial tree such as that shown in Figure 10.3. Here the stock price starts at $20 and in each of two time steps may go up by 10% or down by 10%. We suppose that each time step is three months long and the risk-free

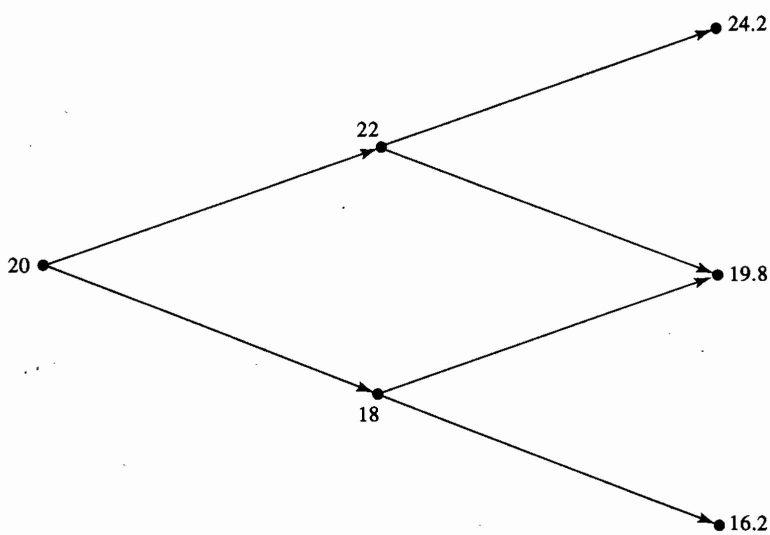

Figure 10.3 Stock prices in a two-step tree

[1] Because the correct value of the option is 0.633, we can deduce that the correct discount rate is 42.58%. This is because $0.633 = 0.7041e^{-0.4258\times 3/12}$.

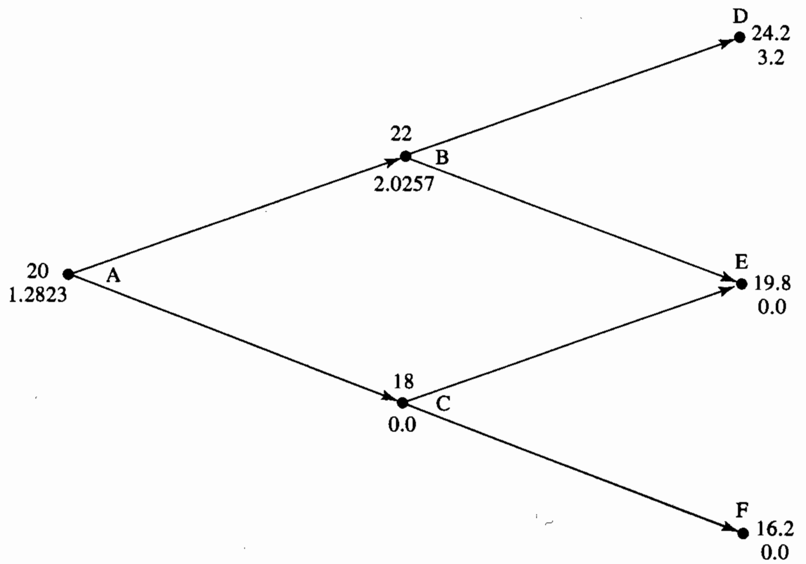

Figure 10.4 Stock and option prices in a two-step tree. The upper number at each node is the stock price; the lower number is the option price

interest rate is 12% per annum. As before, we consider an option with a strike price of \$21.

The objective of the analysis is to calculate the option price at the initial node of the tree. This can be done by repeatedly applying the principles established earlier in the chapter. Figure 10.4 shows the same tree as Figure 10.3, but with both the stock price and the option price at each node. (The stock price is the upper number and the option price is the lower number.) The option prices at the final nodes of the tree are easily calculated. They are the payoffs from the option. At node D the stock price is 24.2 and the option price is $24.2 - 21 = 3.2$; at nodes E and F the option is out of the money and its value is zero.

At node C the option price is zero, because node C leads to either node E or node F and at both nodes the option price is zero. We calculate the option price at node B by focusing our attention on the part of the tree shown in Figure 10.5. Using the notation introduced earlier in the chapter, $u = 1.1$, $d = 0.9$, $r = 0.12$, and $T = 0.25$ so that $p = 0.6523$ and equation (10.2) gives the value of the option at node B as

$$e^{-0.12\times 3/12}(0.6523 \times 3.2 + 0.3477 \times 0) = 2.0257$$

It remains for us to calculate to option price at the initial node A. We do so by focusing on the first step of the tree. We know that the value of the option at node B is 2.0257 and that at node C it is zero. Equation (10.2) therefore gives the value at node A as

$$e^{-0.12\times 3/12}(0.6523 \times 2.0257 + 0.3477 \times 0) = 1.2823$$

The value of the option is \$1.2823.

Note that this example was constructed so that u and d (the proportional up and down movements) were the same at each node of the tree and so that the time steps were of the same length. As a result, the risk-neutral probability, p, as calculated by equation (10.3) is the same at each node.

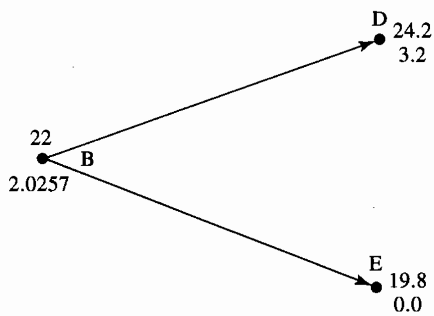

Figure 10.5 Evaluation of option price at node B

A Generalization

We can generalize the case of two time steps by considering the situation in Figure 10.6. The stock price is initially S_0. During each time step, it either moves up to u times its initial value or moves down to d times its initial value. The notation for the value of the option is shown on the tree. (For example, after two up movements the value of the option is f_{uu}.) We suppose that the risk-free interest rate is r and the length of the time step is δt years.

Repeated application of equation (10.2) gives

$$f_u = e^{-r\delta t}[pf_{uu} + (1-p)f_{ud}] \tag{10.5}$$

$$f_d = e^{-r\delta t}[pf_{ud} + (1-p)f_{dd}] \tag{10.6}$$

$$f = e^{-r\delta t}[pf_u + (1-p)f_d] \tag{10.7}$$

Substituting from equations (10.5) and (10.6) into (10.7), we get

$$f = e^{-2r\delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}] \tag{10.8}$$

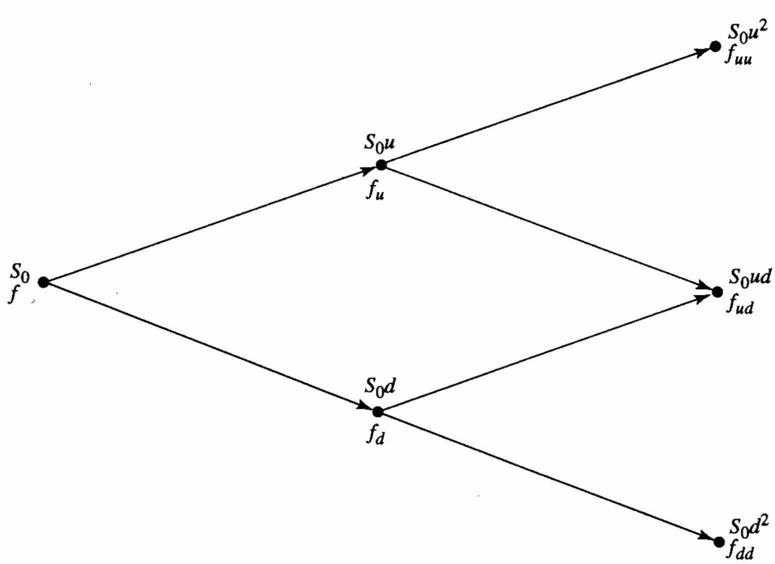

Figure 10.6 Stock and option prices in general two-step tree

This is consistent with the principle of risk-neutral valuation mentioned earlier. The variables p^2, $2p(1-p)$, and $(1-p)^2$ are the probabilities that the upper, middle, and lower final nodes will be reached. The option price is equal to its expected payoff in a risk-neutral world discounted at the risk-free interest rate.

As we add more steps to the binomial tree, the risk-neutral valuation principle continues to hold. The option price is always equal to its expected payoff in a risk-neutral world, discounted at the risk-free interest rate.

10.4 A PUT EXAMPLE

The procedures described in this chapter can be used to price any derivative dependent on a stock whose price changes are binomial. Consider a two-year European put with a strike price of \$52 on a stock whose current price is \$50. We suppose that there are two time steps of one year, and in each time step the stock price either moves up by a proportional amount of 20% or moves down by a proportional amount of 20%. We also suppose that the risk-free interest rate is 5%.

The tree is shown in Figure 10.7. The value of the risk-neutral probability, p, is given by

$$p = \frac{e^{0.05\times 1} - 0.8}{1.2 - 0.8} = 0.6282$$

The possible final stock prices are: \$72, \$48, and \$32. In this case $f_{uu} = 0$, $f_{ud} = 4$, and $f_{dd} = 20$. From equation (10.8)

$$f = e^{-2\times 0.05\times 1}(0.6282^2 \times 0 + 2 \times 0.6282 \times 0.3718 \times 4 + 0.3718^2 \times 20) = 4.1923$$

The value of the put is \$4.1923. This result can also be obtained using equation (10.2)

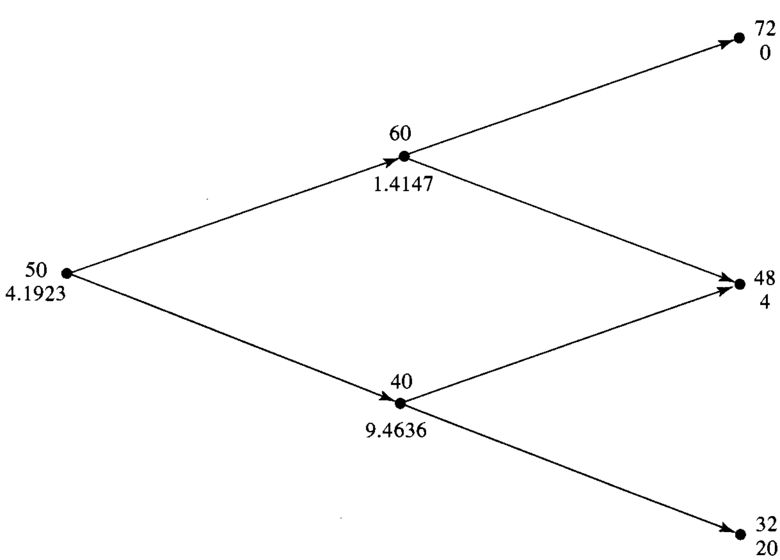

Figure 10.7 Use of two-step tree to value European put option. At each node the upper number is the stock price; the lower number is the option price

and working back through the tree one step at a time. Figure 10.7 shows the intermediate option prices that are calculated.

10.5 AMERICAN OPTIONS

Up to now all the options we have considered have been European. We now move on to consider how American options can be valued using a binomial tree such as that in Figures 10.4 or 10.7. The procedure is to work back through the tree from the end to the beginning, testing at each node to see whether early exercise is optimal. The value of the option at the final nodes is the same as for the European option. At earlier nodes the value of the option is the greater of

1. The value given by equation (10.2)
2. The payoff from early exercise

Figure 10.8 shows how Figure 10.7 is affected if the option under consideration is American rather than European. The stock prices and their probabilities are unchanged. The values for the option at the final nodes are also unchanged. At node B, equation (10.2) gives the value of the option as 1.4147, whereas the payoff from early exercise is negative (= −8). Clearly early exercise is not optimal at node B, and the value of the option at this node is 1.4147. At node C, equation (10.2) gives the value of the option as 9.4636, whereas the payoff from early exercise is 12. In this case, early exercise is optimal and the value of the option at the node is 12. At the initial node A, the value given by equation (10.2) is

$$e^{-0.05\times 1}(0.6282 \times 1.4147 + 0.3718 \times 12.0) = 5.0894$$

and the payoff from early exercise is 2. In this case early exercise is not optimal. The value of the option is therefore $5.0894.

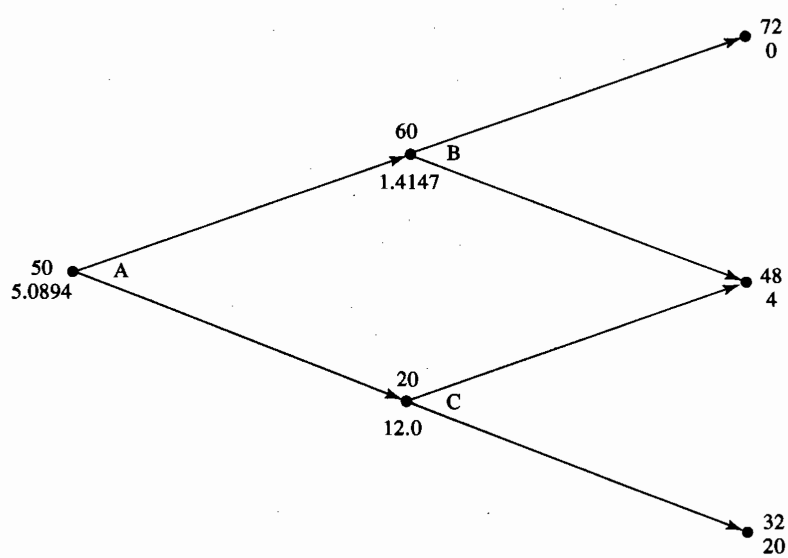

Figure 10.8 Use of two-step tree to value American put option. At each node the upper number is the stock price; the lower number is the option price

More details on the use of binomial trees to value American options are given in Chapter 17.

10.6 DELTA

At this stage it is appropriate to discuss *delta*, an important parameter in the pricing and hedging of options.

The delta of a stock option is the ratio of the change in the price of the stock option to the change in the price of the underlying stock. It is the number of units of the stock we should hold for each option shorted in order to create a riskless hedge. It is the same as the Δ introduced earlier in this chapter. The construction of a riskless hedge is sometimes referred to as *delta hedging*. The delta of a call option is positive, whereas the delta of a put option is negative.

From Figure 10.1, we can calculate the value of the delta of the call option being considered as

$$\frac{1-0}{22-18}=0.25$$

This is because when the stock price changes from \$18 to \$22, the option price changes from \$0 to \$1.

In Figure 10.4 the delta corresponding to stock price movements over the first time step is

$$\frac{2.0257-0}{22-18}=0.5064$$

The delta for stock price movements over the second time step is

$$\frac{3.2-0}{24.2-19.8}=0.7273$$

if there is an upward movement over the first time step and

$$\frac{0-0}{19.8-16.2}=0$$

if there is a downward movement over the first time step.

From Figure 10.7, delta is

$$\frac{1.4147-9.4636}{60-40}=-0.4024$$

at the end of the first time step and either

$$\frac{0-4}{72-48}=-0.1667$$

or

$$\frac{4-20}{48-32}=-1.0000$$

at the end of the second time step.

The two-step examples show that delta changes over time. (In Figure 10.4 delta changes from 0.5064 to either 0.7273 or 0; in Figure 10.7 it changes from −0.4024 to either −0.1667 or −1.0000.) Thus, in order to maintain a riskless hedge using an option and the underlying stock, we need to adjust our holdings in the stock periodically. This is a feature of options that we will return to in Chapters 11 and 15.

10.7 BINOMIAL TREES IN PRACTICE

The binomial models presented so far have been unrealistically simple. Clearly an analyst can expect to obtain only a very rough approximation to an option price by assuming that stock price movements during the life of the option consist of one or two binomial steps.

When binomial trees are used in practice, the life of the option is typically divided into 30 or more time steps. In each time step there is a binomial stock price movement. With 30 time steps this means that 31 terminal stock prices and 2^{30}, or about 1 billion, possible stock price paths are considered.

The values of u and d are determined from the stock price volatility, σ. The Cox, Ross, and Rubinstein method sets

$$u = e^{\sigma\sqrt{\delta t}}$$

and

$$d = \frac{1}{u}$$

The complete set of equations defining the tree is then

$$u = e^{\sigma\sqrt{\delta t}}, \qquad d = e^{-\sigma\sqrt{\delta t}}$$

$$p = \frac{e^{r\delta t} - d}{u - d}$$

Chapter 17 provides a further discussion of these formulas and practical issues involved in the construction and use of binomial trees.

10.8 SUMMARY

This chapter has provided a first look at the valuation of stock options. If stock price movements during the life of an option are governed by a one-step binomial tree, it is possible to set up a portfolio consisting of a stock option and the stock that is riskless. In a world with no arbitrage opportunities, riskless portfolios must earn the risk-free interest. This enables the stock option to be priced in terms of the stock. It is interesting to note that no assumptions are required about the probabilities of up and down movements in the stock price at each node of the tree.

When stock price movements are governed by a multistep binomial tree, we can treat each binomial step separately and work back from the end of the life of the option to the beginning to obtain the current value of the option. Again only no-arbitrage arguments are used, and no assumptions are required about the probabilities of up and down movements in the stock price at each node.

Another approach to valuing stock options involves risk-neutral valuation. This very important principle states that it is permissible to assume the world is risk neutral when valuing an option in terms of the underlying stock. This chapter has shown, through both numerical examples and algebra, that no-arbitrage arguments and risk-neutral valuation are equivalent and lead to the same option prices.

The delta of a stock option, Δ, considers the effect of a small change in the underlying stock price on the change in the option price. It is the ratio of the change in the option price to the change in the stock price. For a riskless position an investor should buy Δ shares for each option sold. An inspection of a typical binomial tree shows that delta changes during the life of an option. This means that to hedge a particular option position, we must change our holding in the underlying stock periodically.

In the next chapter we examine the Black–Scholes analytic approach to pricing stock options. In Chapters 12 and 13 we review other types of options. In Chapter 15 we consider hedge statistics such as delta. In Chapter 17 we return to binomial trees and give a more complete discussion of how they are implemented.

Suggestions for Further Reading

Cox, J., S. Ross, and M. Rubinstein. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7 (October 1979): 229–64.

Rendleman, R., and B. Bartter. "Two State Option Pricing." *Journal of Finance* 34 (1979): 1092–1110.

Quiz (Answers at End of Book)

10.1. A stock price is currently \$40. It is known that at the end of one month it will be either \$42 or \$38. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-month European call option with a strike price of \$39?

10.2. Explain the no-arbitrage and risk-neutral valuation approaches to valuing a European option using a one-step binomial tree.

10.3. What is meant by the delta of a stock option?

10.4. A stock price is currently \$50. It is known that at the end of six months it will be either \$45 or \$55. The risk-free interest rate is 10% per annum with continuous compounding. What is the the value of a six-month European put option with a strike price of \$50?

10.5. A stock price is currently \$100. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of \$100?

10.6. For the situation considered in Question 10.5, what is the value of a one-year European put option with a strike price of \$100? Verify that the European call and European put prices satisfy put–call parity.

10.7. Consider the situation in which stock price movements during the life of a European option are governed by a two-step binomial tree. Explain why it is not possible to set up a position in the stock and the option that remains riskless for the whole of the life of the option.

Questions and Problems (Answers in Solutions Manual)

10.8. A stock price is currently \$50. It is known that at the end of two months it will be either \$53 or \$48. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a two-month European call option with a strike price of \$49? Use no-arbitrage arguments.

10.9. A stock price is currently \$80. It is known that at the end of four months it will be either \$75 or \$85. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a four-month European put option with a strike price of \$80? Use no-arbitrage arguments.

10.10. A stock price is currently \$40. It is known that at the end of three months it will be either \$45 or \$35. The risk-free rate of interest with quarterly compounding is 8% per annum. Calculate the value of a three-month European put option on the stock with an exercise price of \$40. Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers.

10.11. A stock price is currently \$50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of \$51?

10.12. For the situation considered in Problem 10.11, what is the value of a six-month European put option with a strike price of \$51? Verify that the European call and European put prices satisfy put–call parity. If the put option were American, would it ever be optimal to exercise it early at any of the nodes on the tree?

10.13. A stock price is currently \$25. It is known that at the end of two months it will be either \$23 or \$27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of two months. What is the value of a derivative that pays off S_T^2 at this time?

Assignment Questions

10.14. A stock price is currently \$50. It is known that at the end of six months it will be either \$60 or \$42. The risk-free rate of interest with continuous compounding is 12% per annum. Calculate the value of a six-month European call option on the stock with an exercise price of \$48. Verify that no-arbitrage arguments and risk-neutral valuation arguments give the same answers.

10.15. A stock price is currently \$40. Over each of the next two three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per annum with continuous compounding.

a. What is the value of a six-month European put option with a strike price of \$42?

b. What is the value of a six-month American put option with a strike price of \$42?

10.16. Using a "trial-and-error" approach, estimate how high the strike price has to be in Problem 10.16 for it to be optimal to exercise the option immediately.