CHAPTER 12 Options on Stock Indices and Currencies

In this chapter we tackle the problem of valuing options on stock indices and currencies. As a first step, some of the results in Chapters 8, 10, and 11 are extended to cover European options on a stock paying a known dividend yield. It is then argued that both stock indices and currencies are analogous to stocks paying dividend yields. This enables the results for options on a stock paying a dividend yield to be applied to these types of options as well.

12.1 A SIMPLE RULE

In this section we produce a simple rule that enables results produced for European options on a non-dividend-paying stock to be extended so that they apply to European options on a stock paying a known dividend yield.

Consider the difference between a stock that pays a dividend yield at a rate q per annum and a similar stock that pays no dividends. Both stocks should provide the same overall return (dividends plus capital gain). The payment of a dividend causes a stock price to drop by an amount equal to the dividend. The payment of a dividend yield at rate q therefore causes the growth rate in the stock price to be less than it would otherwise be by an amount q. If, with a dividend yield of q, the stock price grows from S_0 today to S_T at time T, then in the absence of dividends it would grow from S_0 today to $S_T e^{qT}$ at time T. Alternatively, in the absence of dividends it would grow from $S_0 e^{-qT}$ today to S_T at time T.

This argument shows that we get the same probability distribution for the stock price at time T in each of the following two cases:

1. The stock starts at price S_0 and pays a dividend yield at rate q.
2. The stock starts at price $S_0 e^{-qT}$ and pays no dividend yield.

This leads to a simple rule. When valuing a European option lasting for time T on a stock paying a known dividend yield at rate q, we reduce the current stock price from S_0 to $S_0 e^{-qT}$, and then value the option as though the stock pays no dividends.

Lower Bounds for Option Prices

As a first application of this rule, consider the problem of determining bounds for the price of a European option on a stock paying a dividend yield at rate q. Substituting S_0e^{-qT} for S_0 in equation (8.1), we see that a lower bound for the European call option price, c, is given by

$$c \geqslant S_0e^{-qT} - Xe^{-rT} \tag{12.1}$$

We can also prove this directly by considering the following two portfolios:

Portfolio A: one European call option plus an amount of cash equal to Xe^{-rT}

Portfolio B: e^{-qT} shares with dividends being reinvested in additional shares

In portfolio A the cash, if it is invested at the risk-free interest rate, will grow to X at time T. If $S_T > X$, the call option is exercised at time T and portfolio A is worth S_T. If $S_T < X$, the call option expires worthless, and the portfolio is worth X. Hence, at time T portfolio A is worth

$$\max(S_T, X)$$

Because of the reinvestment of dividends, portfolio B becomes one share at time T. It is therefore worth S_T at this time. It follows that portfolio A is always worth as much as, and is sometimes worth more than, portfolio B at time T. In the absence of arbitrage opportunities, this must also be true today. Hence,

$$c + Xe^{-rT} \geqslant S_0e^{-qT}$$

or

$$c \geqslant S_0e^{-qT} - Xe^{-rT}$$

To obtain a lower bound for a European put option, we can similarly replace S_0 by S_0e^{-qT} in equation (8.2) to get

$$p \geqslant Xe^{-rT} - S_0e^{-qT} \tag{12.2}$$

This result can also be proved directly by considering

Portfolio C: one European put option plus e^{-qT} shares with dividends on the shares being reinvested in additional shares

Portfolio D: an amount of cash equal to Xe^{-rT}

Put–Call Parity

Replacing S_0 by S_0e^{-qT} in equation (8.3) we obtain put–call parity for an option on a stock paying a dividend yield at rate q:

$$c + Xe^{-rT} = p + S_0e^{-qT} \tag{12.3}$$

This result can also be proved directly by considering the following two portfolios:

Portfolio A: one European call option plus an amount of cash equal to Xe^{-rT}

Portfolio C: one European put option plus e^{-qT} shares with dividends on the shares being reinvested in additional shares

Both portfolios are both worth $\max(S_T, X)$ at time T. They must therefore be worth the same today, and the put–call parity result in equation (12.3) follows. For American options, the put–call parity relationship is (see Problem 12.12)

$$S_0e^{-qT} - X \leqslant C - P \leqslant S_0 - Xe^{-rT}$$

12.2 PRICING FORMULAS

By replacing S_0 by S_0e^{-qT} in the Black–Scholes formulas, equations (11.5) and (11.6), we obtain the price, c, of a European call and the price, p, of a European put on a stock paying a dividend yield at rate q as:

$$c = S_0e^{-qT}N(d_1) - Xe^{-rT}N(d_2) \tag{12.4}$$

$$p = Xe^{-rT}N(-d_2) - S_0e^{-qT}N(-d_1) \tag{12.5}$$

Because

$$\ln\frac{S_0e^{-qT}}{X} = \ln\frac{S_0}{X} - qT$$

d_1 and d_2 are given by

$$d_1 = \frac{\ln(S_0/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

and

$$d_2 = \frac{\ln(S_0/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

These results were first derived by Merton.[1] As discussed in Chapter 11, the word *dividend* should, for the purposes of option valuation, be defined as the reduction in the stock price on the ex-dividend date arising from any dividends declared. If the dividend yield rate is known but not constant during the life of the option, equations (12.4) and (12.5) are still true, with q equal to the average annualized dividend yield.

12.3 BINOMIAL TREES

We now move on to examine the effect of a dividend yield equal to q on the results for the binomial model in Chapter 10.

Consider the situation in Figure 12.1, in which a stock price starts at S_0 and moves either up to S_0u or down to S_0d. As in Chapter 10, we define p as the probability of an up movement in a risk-neutral world. The total return provided by the stock in a risk-neutral world must be the risk-free interest rate, r. The dividends provide a return equal to q. The return in the form of capital gains must be $r - q$. This means that p must satisfy

$$pS_0u + (1 - p)S_0d = S_0e^{(r-q)T} \tag{12.6}$$

or

$$p = \frac{e^{(r-q)T} - d}{u - d} \tag{12.7}$$

[1] See Merton, R. C. "Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science* 4 (spring 1973): 141–83.

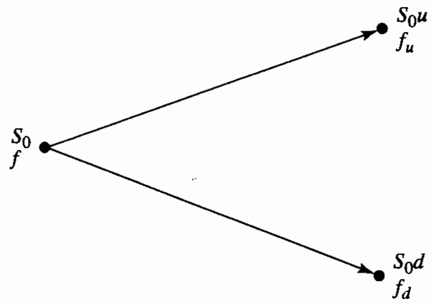

Figure 12.1 Stock price and option price in one-step binomial tree

As noted in Chapter 10, the value of the derivative at time zero is the expected payoff in a risk-neutral world discounted at the risk-free rate:

$$f = e^{-rT}[pf_u + (1-p)f_d] \tag{12.8}$$

Example

Suppose that the initial stock price is \$30 and the stock price will either move up to \$36 or down to \$24 during a six-month period. The six-month risk-free interest rate is 5%, and the stock is expected to provide a dividend yield of 3% during the six-month period. In this case $u = 1.2$, $d = 0.8$, and

$$p = \frac{e^{(0.05-0.03)\times 6/12} - 0.8}{1.2 - 0.8} = 0.5251$$

Consider a six-month put option on the stock with a strike price of \$28. If the stock price moves up, the payoff is zero; if it moves down, the payoff is 4. The value of the option is therefore

$$e^{-0.05\times 0.5}[0.5251 \times 0 + 0.4749 \times 4] = 1.85$$

12.4 OPTIONS ON STOCK INDICES

As discussed in Chapter 7, several exchanges trade options on stock indices. Some of the indices track the movement of the market as a whole. Others are based on the performance of a particular sector (e.g., computer technology, oil and gas, transportation, or telephone).

Quotes

Table 12.1 shows quotes for options on the Dow Jones Industrial Average (DJX), Nasdaq (NDX), Russell 2000 (RUT), S&P 100 (OEX), and S&P 500 (SPX) index options as they appeared in the Money and Investing section of the *Wall Street Journal* on Friday March 16, 2001. All the options trade on the Chicago Board Options Exchange and all are European, except the contract on the S&P 100, which is American. The quotes refer to the price at which the last trade was made on Thursday March 15, 2001. The closing prices of the DJX, NDX, RUT, OEX, and SPX on March 15, 2001 were 100.31, 1,697.92, 452.16, 600.71, and 1,173.56, respectively.

Table 12.1 Quotes for stock index options from *The Wall Street Journal*, March 16, 2001

Thursday, March 15, 2001

Volume, last, net change and open interest for all contracts. Volume figures are unofficial. Open interest reflects previous trading day. p-Put c-Call

Strike		Vol.	Last	Net Chg.	Open Int.
CHICAGO					
DJ INDUS AVG(DJX)					
Jun	72 p	57	0^{30}	-0^{05}	115
Jun	80 p	10	0^{85}	-0^{15}	4,999
Apr	84 p	10	0^{25}	...	...
Jun	84 p	20	1	-0^{10}	1,016
Jun	88 p	5	1^{25}	-0^{15}	227
Apr	90 p	40	0^{50}	...	...
Jun	90 p	126	1^{60}	...	5,080
Sep	90 p	93	2^{60}	-0^{15}	203
Apr	92 p	1,787	0^{85}	-0^{15}	5,604
Jun	92 p	45	1^{90}	-0^{25}	2,381
Sep	92 p	59	3	$+0^{35}$	503
Mar	96 c	8	4^{50}	$+0^{40}$	114
Mar	96 p	170	0^{05}	-0^{15}	12,199
Apr	96 c	6	5^{80}	-0^{90}	3
Apr	96 p	572	1^{75}	-0^{10}	5,670
Jun	96 c	3	7^{80}	-1^{90}	489
Jun	96 p	628	3^{10}	-0^{40}	4,987
Sep	96 p	23	4^{20}	...	6,374
Mar	98 c	20	1^{90}	-0^{10}	32
Mar	98 p	748	0^{05}	-0^{35}	1,502
Apr	98 c	49	4^{40}	-0^{10}	8,116
Apr	98 p	836	2^{20}	...	8,469
Mar	100 c	2,323	0^{70}	-0^{30}	3,849
Mar	100 p	4,550	0^{30}	-0^{70}	14,425
Apr	100 c	463	3^{60}	$+0^{20}$	8,549
Apr	100 p	1,502	2^{85}	-0^{15}	10,208
Jun	100 c	1,125	5	...	1,292
Jun	100 p	3,119	4^{10}	-0^{30}	7,207
Sep	100 c	2	7^{50}	$+0^{60}$	30
Sep	100 p	934	5^{70}	...	4,295
Mar	102 c	478	0^{10}	-0^{15}	3,901
Mar	102 p	1,638	1^{85}	-0^{60}	5,648
Apr	102 c	148	2^{60}	-0^{05}	629
Apr	102 p	103	3^{80}	-0^{40}	9,441
Jun	102 c	29	4^{10}	-0^{40}	2,295
Jun	102 p	513	5	$+0^{70}$	2,313
Mar	103 c	49	0^{05}	-0^{15}	470
Mar	103 p	305	2^{80}	-0^{30}	1,577
Mar	104 p	740	3^{80}	-0^{70}	8,693
Apr	104 c	77	1^{60}	$+0^{10}$	1,698
Apr	104 p	197	5^{10}	-0^{50}	13,625
Jun	104 c	2	3^{70}	$+0^{50}$	2,484
Jun	104 p	230	6^{20}	$+0^{20}$	5,149
Sep	104 p	1	7^{40}	$+0^{40}$	996
Mar	105 p	127	4^{80}	-0^{10}	1,536
Mar	106 p	181	6	$+0^{30}$	5,792
Apr	106 c	637	1	-0^{20}	679
Apr	106 p	41	6^{10}	$+0^{30}$	43,208
Jun	106 c	82	2^{50}	$+0^{20}$	243
Jun	106 p	61	7^{30}	$+0^{30}$	698
Mar	107 p	2	6^{80}	-0^{10}	652
Mar	108 p	214	8	...	3,962
Apr	108 c	15	0^{95}	$+0^{40}$	1,226
Apr	108 p	20	8	-0^{80}	70,397
Jun	108 p	53	8^{40}	-0^{80}	1,917
Sep	108 p	11	9^{30}	$+0^{30}$	746
Mar	110 p	107	9^{50}	-0^{10}	8,010
Apr	110 c	1,022	0^{35}	...	7,752
Apr	110 p	58	9^{50}	-0^{10}	376
Jun	110 p	34	9^{80}	-0^{40}	1,253
Mar	112 p	11	12	-1^{40}	1,166
Apr	112 c	900	0^{25}	$+0^{05}$	5,217
Jun	112 p	1	10^{10}	-1^{90}	622
Sep	112 p	1	12	-0^{20}	390
Call Vol.		9,262	Open Int.		178,509
Put Vol.		22,787	Open Int.		361,587
NASDAQ-100(NDX)					
Apr	1200 p	17	7	-1^{10}	980
May	1200 p	17	18	-1	891
Apr	1300 p	26	12	-6	144
Apr	1400 p	62	29^{10}	-0^{90}	1,827
May	1400 p	45	48	-4	1,261
Apr	1500 p	104	53	$+3$	517
Mar	1550 p	20	0^{25}	-1^{75}	252
Apr	1550 p	6	42	-16^{40}	228
Mar	1600 p	665	1^{25}	-2^{95}	603
Apr	1600 p	51	58	-12	940
Jun	1600 p	1	130^{40}	$+10^{40}$	832
Mar	1650 c	17	42^{20}	-46^{80}	...
Mar	1650 p	388	15	-1^{20}	1,284
Apr	1650 p	2	75	-12	597
Mar	1700 c	217	20	-55	263
Mar	1700 p	1,358	25	...	2,149
Apr	1700 p	10	133	-1^{80}	844
May	1700 p	10	135	-10	21
Jun	1700 c	2	203	-17	26
Jun	1700 p	3	180	...	799
Mar	1750 c	105	4	-30^{40}	157
Mar	1750 p	191	65	$+23^{20}$	541
Apr	1750 c	6	110	-15	6
Apr	1750 p	363	152	$+30$	30
Mar	1800 c	248	1	-20^{90}	330
Mar	1800 p	209	100	$+15$	784
Apr	1800 c	1	120	$+23$	87
Apr	1800 p	80	170	$+15$	421
Jun	1800 c	3	195	$+15$	36
Jun	1800 p	16	230	$+8$	929
Mar	1850 c	536	1^{25}	-7^{25}	177
Mar	1850 p	2	158^{30}	$+33^{30}$	227
Apr	1850 c	29	73^{80}	-38^{20}	118
Apr	1850 p	6	199	-1	91
Mar	1900 c	359	0^{50}	-3	520
Mar	1900 p	39	190	$+80$	1,270
Apr	1900 c	292	69^{30}	-18^{50}	838
Apr	1900 p	56	241	$+16$	120
May	1900 c	1	142	$+33^{80}$	44
Jun	1900 c	26	120	-30	163
Mar	1950 c	113	0^{10}	-1^{85}	725
Mar	1950 p	82	200	$+30$	1,662
Apr	1950 c	5	80	$+15$	767
Apr	1950 p	23	245	-5	525
Mar	2050 c	10	0^{25}	-0^{25}	578
Mar	2050 p	103	350	$+85$	2,315
Apr	2050 c	280	30	-11^{50}	271
Apr	2050 p	107	308	-34^{50}	52
May	2050 c	1	86	-2	461
Mar	2100 p	3	395	$+45$	1,439
Apr	2100 c	240	33	-1	462
Apr	2100 p	100	395	$+15$	305
Jun	2100 c	1	78^{60}	-31^{40}	51
Mar	2150 c	1	0^{20}	-0^{30}	425
Mar	2150 p	98	370	$+70$	256
Apr	2150 p	4	455	$+40^{30}$	14
Mar	2200 c	6	0^{15}	-0^{35}	1,356
Mar	2200 p	12	435	-60^{30}	579
Apr	2200 c	3	15^{60}	-4^{90}	53
Apr	2200 p	1	470	$+185$	18
May	2200 p	300	435	-47	600
Jun	2200 c	1	61	$+5$	61
Jun	2200 p	52	455	$+240$	283
Mar	2250 c	15	0^{05}	-0^{05}	412
Mar	2250 p	7	544	$+26$	170
Apr	2250 c	57	10	-8^{20}	13
Mar	2300 c	3	1^{50}	$+1$	1,164
Mar	2300 p	1	525	$+237$	1,138
Apr	2300 c	23	7	-3^{80}	438
Apr	2300 p	4	583^{10}	$+188$	240
Jun	2300 p	1	555^{90}	$+0^{90}$	93
Mar	2350 p	51	659^{40}	$+220$	575
Mar	2400 p	5	685	$+10$	1,338
Apr	2400 c	2	8^{60}	$+2^{80}$	163
Jun	2400 c	27	33	-2	1,016
Jun	2400 p	4	615^{90}	-68^{10}	500
Apr	2450 p	2	685	-25	...
Mar	2500 p	3	805	$+5$	542
Apr	2500 c	12	2^{70}	-1^{80}	1,268
May	2500 c	39	9^{50}	...	2
May	2500 p	3	696^{20}	...	...
Mar	2600 c	10	0^{05}	-0^{45}	760
Mar	2600 p	8	900	$+66^{80}$	785
Apr	2600 c	5	1^{70}	-0^{95}	799
Apr	2600 p	3	805	-30	87
Mar	2700 p	1	910	-75	332
Apr	2800 c	1	1	-0^{30}	2,726
May	2900 c	7	3	$+0^{50}$	216
Jun	2900 c	7	7	$+1^{10}$	1,067
Apr	2950 c	12	0^{70}	-2^{40}	973
Jun	3050 c	10	3	$+0^{30}$	143
Call Vol.		2,732	Open Int.		66,026
Put Vol.		4,605	Open Int.		52,260
RUSSELL 2000(RUT)					
Mar	450 c	10	3^{60}	-31^{70}	10
Mar	450 p	460	1^{40}	-0^{85}	1,243
Apr	450 p	1,644	17	-3	500
Mar	460 c	15	1^{90}	-4^{90}	406
Mar	460 p	29	9^{10}	$+2^{90}$	498
Apr	460 c	100	13^{50}	-2^{50}	651
Apr	460 p	201	22^{80}	$+1^{20}$	401
Mar	470 c	10	0^{50}	-9^{80}	11
Apr	470 p	500	24^{50}	$+1$	1,000
Mar	500 p	100	48	$+2^{90}$	456
May	500 c	10	7^{50}	-1^{10}	298
Apr	510 c	13	3	...	...
Call Vol.		158	Open Int.		7,706
Put Vol.		2,397	Open Int.		15,139
S & P 100(OEX)					
Apr	500 p	311	2	-0^{70}	1,515
May	500 p	24	3^{50}	-1	186
Jun	500 p	1	5^{50}	-0^{30}	180
Apr	510 p	14	2^{75}	-0^{95}	1,233
Mar	540 p	847	0^{10}	-0^{15}	3,610
Apr	540 p	288	5	-1^{70}	4,208
Jun	540 p	10	11^{10}	-2^{40}	538
Mar	550 c	5	52	$+6^{10}$	7
Mar	550 p	749	0^{05}	-0^{45}	5,503
Apr	550 p	147	7^{10}	-1^{30}	806
Apr	555 p	25	7^{20}	...	...
Mar	580 p	857	0^{10}	-0^{70}	8,327
Apr	580 c	1	55^{50}	...	...
Apr	560 p	124	8^{20}	-1^{10}	2,851
Jun	580 p	122	15^{10}	$+1^{10}$	794
Mar	570 p	2,136	0^{30}	-1^{30}	4,696
Apr	570 c	10	47	$+6^{80}$	10
Apr	570 p	159	10^{70}	-1^{60}	1,802
Apr	575 p	4	11^{50}	...	...
Mar	580 p	2,164	1	-1^{50}	7,552
Apr	580 p	320	13^{40}	-0^{50}	3,252
May	580 p	14	18^{70}	-1^{30}	3,418
Jun	580 c	122	48^{50}	-1^{50}	5
Jun	580 p	123	21	-3^{40}	248
Apr	585 p	1	14^{30}	...	...
Mar	590 c	157	11	-1	772
Mar	590 p	3,264	1^{70}	-3^{10}	3,765
Apr	590 c	26	29^{90}	$+1^{40}$	34
Apr	590 p	46	15	-3^{50}	1,431
May	590 p	2	21	...	...
Apr	595 p	27	18^{20}	...	...
Mar	600 c	6,508	5^{70}	-1^{20}	3,563
Mar	600 p	8,644	4^{70}	-4^{30}	7,950
Apr	600 c	393	23^{90}	$+2^{80}$	465
Apr	600 p	828	20^{50}	-3^{30}	3,411
May	600 p	105	24	-5	370
Jun	600 c	1	35^{40}	$+0^{10}$	3
Jun	600 p	19	28	-2^{50}	316
Mar	605 c	4,300	2^{35}	-1^{95}	2,019
Mar	605 p	1,901	7^{30}	-4^{70}	2,035
Apr	605 c	281	20	$+0^{10}$	47
Apr	605 p	23	23^{30}	-1^{70}	165
Mar	610 c	6,920	1^{75}	-1^{25}	4,489
Mar	610 p	2,738	10	-6	2,208
Apr	610 c	98	17^{10}	...	191
Apr	610 p	95	25^{30}	-3^{20}	714
May	610 p	4	31^{50}	...	...
Mar	615 c	1,773	0^{80}	-1^{40}	3,267
Mar	615 p	662	20^{80}	$+0^{60}$	2,077

(*continued on next page*)

Table 12.1 *(continued)*

Apr	615 c	32	15	+	050	332
Apr	615 p	2	28	–	5	315
Mar	620 c	4,149	030	–	1	3,881
Mar	620 p	1,807	2080	–	420	3,965
Apr	620 c	352	1320	+	1	599
Apr	620 p	131	2910	–	490	3,294
May	620 c	157	1930	+	130	25
May	620 p	10	3510	–	490	99
Jun	620 c	6	26	–	050	885
Jun	620 p	8	3750	–	340	1,159
Mar	625 c	1,281	015	–	045	1,792
Mar	625 p	55	24	–	5	810
Apr	625 c	79	1120	+	020	180
Apr	625 p	7	3030	+	430	182
Mar	630 c	1,874	005	–	035	3,458
Mar	630 p	928	29	–	5	2,106
Apr	630 c	156	950	+	030	870
Apr	630 p	146	3520	–	430	1,355
May	630 c	1,187	16	+	2	16
May	630 p	9	40	–	3	331
Mar	635 c	1,313	005	–	015	2,697
Mar	635 p	456	3520	–	280	735
Apr	635 c	15	820	+	070	60
Apr	635 p	5	3850	+	350	644
Mar	640 c	488	005	–	010	4,378
Mar	640 p	110	40	–	5	634
Apr	640 c	93	650	–	020	951
Apr	640 p	80	4340	–	960	711
May	640 c	8	1110	–	020	64
May	640 p	22	4730	–	370	396
Jun	640 c	24	1550	+	1	164
Jun	640 p	12	4850	–	450	614
Mar	645 c	265	010	–	005	2,420
Mar	645 p	147	4430	–	420	1,227
Apr	645 c	41	5	–	1	448
Apr	645 p	48	4750	+	9	166
Mar	650 c	121	005	–	005	5,151
Mar	650 p	129	5090	–	410	407
Apr	650 c	300	420	–	060	2,160
Apr	650 p	32	5250	–	4	2,016
May	650 c	27	920	+	120	169
Mar	655 p	23	55	–	5	108
Apr	655 c	73	350	+	040	259
Apr	655 p	9	54	–	250	435
Mar	660 c	1,022	005		...	4,351
Mar	660 p	42	59	–	6	386
Apr	660 c	111	310	–	050	2,959
Apr	660 p	169	6110	–	390	456
May	660 c	220	650	–	2	178
Mar	665 c	65	005		...	4,248
Mar	665 p	1	61	–	9	94
Apr	665 c	150	240	–	025	175
Mar	670 c	95	005		...	4,657
Mar	670 p	641	69	–	8	765
Apr	670 c	185	180	–	040	704
Apr	670 p	45	7250	+	350	447
May	670 p	13	70	+	3	138
Mar	675 c	401	005		...	3,505
Mar	675 p	365	7430	–	670	706
Apr	675 c	81	155	–	045	459
Apr	675 p	50	71	–	3	7
Mar	680 p	108	7850	–	650	150
Apr	680 c	85	130	–	015	1,587
Apr	680 p	19	79	–	820	533
May	680 c	327	350	+	070	63
Jun	680 p	10	78	–	9	503
Apr	690 c	248	095	–	005	3,011
Apr	690 p	109	90	–	10	353
May	690 c	291	230	–	010	186
May	690 p	10	90	+	680	47
Mar	700 p	25	9850	–	450	66
Apr	700 c	65	080	–	035	2,933
Apr	700 p	9	9850	–	920	953
Apr	705 c	30	055	+	005	389
Apr	705 p	10	109		...	...
Apr	720 c	231	020	–	015	1,024
Apr	720 p	9	116	–	580	149
Apr	725 p	11	12010	+	4610	12
Apr	730 c	125	025	–	020	837
May	730 p	14	12570	+	48	25
Jun	740 p	60	138	–	170	50
May	750 p	16	14570	+	4910	7
May	760 c	4	035	–	075	210
May	760 p	3	15590	+	850	4
Call Vol.		35,250	Open Int.			171,210
Put Vol.		34,927	Open Int.			117,069
		S & P 500(SPX)				
Jun	750 c	300	432	–	195	306
Jun	750 p	14	090	–	010	6,555
Jun	800 p	161	140	–	035	3,039
Jun	850 p	1	270	–	030	924
Jun	900 c	1	291	+	11	415
Jun	900 p	79	410	–	060	25,204
Apr	950 p	391	220	–	065	1,974
Jun	950 p	4	740	+	120	6,802
Apr	975 p	1,225	310	–	040	656
Mar	995 c	5	181	–	6010	11,324
Mar	995 p	100	005	–	005	25,234
Jun	995 c	955	198		...	914
Jun	995 p	603	11		...	4,941
Apr	1005 p	641	4	–	110	714
Mar	1025 p	200	005	–	025	8,705
Apr	1025 p	292	6	–	160	4,829
Mar	1050 p	320	005	–	055	9,692
Apr	1050 c	6	133	+	3	8
Apr	1050 p	1,055	8	–	150	6,341
Jun	1050 p	1,488	1850	+	160	6,702
Apr	1075 p	2,394	11	–	030	10,504
Mar	1100 p	842	010	–	070	28,705
Apr	1100 c	1	9250	+	150	57
Apr	1100 p	5,741	14	–	3	8,970
May	1100 p	363	2250	–	250	3,078
Jun	1100 c	301	11340	–	560	216
Jun	1100 p	33	2830	–	220	16,096
Mar	1125 p	1,303	025	–	175	4,529
Apr	1125 p	261	1950	–	350	3,737
May	1125 p	872	2750	–	450	3,996
Jun	1125 c	1,000	9350	+	850	7
Mar	1150 c	908	2280	–	020	1,801
Mar	1150 p	4,934	140	–	490	12,363
Apr	1150 c	2,755	55	+	3	189
Apr	1150 p	4,247	2750	–	250	19,128
May	1150 c	10	69	+	5	105
May	1150 p	7,741	36	–	3	6,656
Jun	1150 c	251	77	–	4	604
Jun	1150 p	712	42	–	5	9,383
Mar	1175 c	2,250	280	–	640	2,960
Mar	1175 p	7,401	6	–	850	13,344
Apr	1175 c	180	3870	+	080	426
Apr	1175 p	2,735	3750	–	4	7,992
May	1175 c	11	54		...	...
May	1175 p	3,716	4520	–	480	5,557
Jun	1175 c	2,122	62	+	2	5,192
Jun	1175 p	3,891	5240	–	060	7,969
Mar	1200 c	5,969	1	–	2	6,286
Mar	1200 p	3,639	28	–	8	16,866
Apr	1200 c	2,131	2550	–	050	5,447
Apr	1200 p	1,265	48	–	4	16,070
May	1200 c	405	41	+	4	1,565
May	1200 p	451	58	–	4	4,572
Jun	1200 c	6,731	51	+	8	7,620
Jun	1200 p	6,344	61	–	650	19,356
Mar	1210 c	651	040	–	085	965
Mar	1210 p	37	33	+	150	1,483
Mar	1220 c	969	050	–	050	12,315
Mar	1220 p	55	4050	+	1	1,962
Mar	1225 c	2,365	015	–	085	12,754
Mar	1225 p	1,667	53	–	5	13,057
Apr	1225 c	1,497	16	–	1	3,307
Apr	1225 p	2,651	62	–	15	6,856
May	1225 c	1,950	30	+	510	120
May	1225 p	2	7020	–	980	1,079
Jun	1225 c	1,004	37	–	4	8,086
Jun	1225 p	34	76	–	8	12,360
Mar	1230 c	250	010	–	065	10,959
Mar	1230 p	60	5650	–	1150	3,435
Mar	1240 c	106	010	–	040	5,757
Mar	1240 p	7	69	+	2	4,573
Mar	1250 c	292	005	–	030	22,662
Mar	1250 p	642	76	–	6	24,292
Apr	1250 c	5,455	10	–	1	11,780
Apr	1250 p	318	7910	–	890	19,663
May	1250 c	1,036	1940	+	130	857
May	1250 p	67	8450	–	850	973
Jun	1250 c	6	2690	–	010	15,503
Jun	1250 p	1	9250	–	9	20,443
Mar	1260 p	55	8750	+	550	5,848
Mar	1270 c	4	005	–	095	1,775
Mar	1275 c	1,943	020		...	25,578
Mar	1275 p	982	103	–	4	22,621
Apr	1275 c	4,461	6	+	090	14,980
Apr	1275 p	380	105	–	10	12,176
May	1275 c	1	1590	+	5	4,304
May	1275 p	3	103	+	1	3,811
Jun	1275 c	1	19	+	250	6,297
Jun	1275 p	331	110	+	5	8,895
Mar	1280 c	2	005		...	3,545
Mar	1280 p	201	104	+	8	700
Mar	1300 p	188	12750	–	850	16,140
Apr	1300 c	438	3	–	030	6,898
Apr	1300 p	5,751	12380	–	1240	6,859
May	1300 c	1,466	850	+	030	4,529
May	1300 p	74	124	–	4	538
Jun	1300 c	624	13	+	150	7,270
Jun	1300 p	117	12550	–	580	13,561
Mar	1310 p	3	13570		...	316
Mar	1325 c	230	010	+	005	47,196
Mar	1325 p	377	148	–	10	42,279
Apr	1325 c	1,139	175	–	030	14,036
Apr	1325 p	211	14650	–	1850	2,854
May	1325 c	25	6		...	273
May	1325 p	70	145	+	1	92
Jun	1325 c	10	10	+	2	5,168
Jun	1325 p	1,002	148	–	10	5,835
Mar	1350 p	174	17720	–	580	38,269
Apr	1350 c	490	1		...	14,129
Apr	1350 p	6,113	173	+	350	1,744
May	1350 c	114	310		...	2,434
Jun	1350 c	505	590	–	170	6,873
Mar	1375 p	132	201	–	7	9,898
Apr	1375 c	140	050	–	025	16,347
Apr	1375 p	51	196	–	450	814
Jun	1375 c	1	450		...	3,788
Mar	1380 p	1	207	+	12	1,703
Mar	1400 p	581	226	–	14	11,624
Apr	1400 p	97	218	+	3	837
May	1400 p	14	216	+	4	512
Jun	1400 p	79	20550	–	1550	8,920
Mar	1425 p	9	252	–	5	10,341
Apr	1425 c	2	030	+	005	9,371
May	1425 p	2	241	+	83	50
Jun	1425 p	125	236	+	4	6,896
Mar	1450 p	36	276	+	7	6,135
Apr	1450 p	3	269	+	150	230
May	1450 c	26	060	–	010	233
May	1450 p	4	265		...	...

LEAPS-LONG TERM

DJ INDUS AVG - CB

Jun 02	100	p	2	770	+	050	243
Dec 02	100	c	15	1660	–	020	79
Dec 02	100	p	35	640	–	1	865
Dec 02	108	c	4	12	–	280	446
Dec 02	108	p	3	1280	+	060	902
Call Vol.			19	Open Int.			8,367
Put Vol.			40	Open Int.			16,890

S & P 100 - CB

Dec 01	190	p	4	190	+	005	490
Call Vol.			0	Open Int.			1,050
Put Vol.			4	Open Int.			5,087

S & P 500 - CB

Dec 01	90	p	13	390	–	050	2,008
Dec 01	100	p	3	310	+	030	1,324
Dec 03	100	p	29	740	+	040	3,026
Dec 01	110	p	9	550		...	21,879
Dec 02	110	p	221	830	+	020	27,363
Dec 03	110	p	10	950	–	040	822
Dec 01	120	p	50	850	–	160	38,210
Dec 02	120	p	5	1170	+	020	4,968
Dec 03	120	p	6	1270	–	090	9,267
Dec 02	12750	p	5	570		...	203
Dec 01	130	p	459	1370	+	010	29,336
Dec 02	130	p	20	1590	–	010	6,690
Dec 03	130	p	37	1690	–	070	12,315
Dec 01	135	p	45	1680	–	1	25,321
Dec 01	140	c	3	245	+	015	7,951
Dec 01	140	p	156	2030	+	010	42,525
Dec 03	140	p	1	2050		...	11,175
Dec 01	145	p	2	2370	+	480	5
Dec 01	145	p	40	2440	+	540	3,430
Dec 01	150	p	95	2870	–	2	3,743
Dec 03	160	p	2	3170		...	11,536
Dec 01	180	c	3	005	–	060	58
Dec 02	180	p	10	50	+	050	13,958
Call Vol.			6	Open Int.			405,282
Put Vol.			1,218	Open Int.			544,821

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One index option contract is on 100 times the index. (Note that the Dow Jones index used for index options is 0.01 times the usually quoted Dow Jones index.) Index options are settled in cash. For example, on exercise of the option, the holder of a call option receives $S - X$ in cash and the writer of the option pays this amount in cash, where S is the value of the index at the close of trading on the day of the exercise and X is the strike price. Similarly, the holder of a put option receives $X - S$ in cash and the writer of the option pays this amount in cash.

Example

Consider the April put option contract on the S&P 100 with a strike price of 620 in Table 12.1. This is an American-style option and expires on April 21, 2001. The cost of one contract is indicated as $29.10 \times 100 = \$2{,}910$. The value of the index at the close of trading on March 15, 2001, is 600.71, so that the option is in the money. If the option contract were exercised, the holder would receive $(620 - 600.71) \times 100 = \$1{,}929$ in cash. This is less than the value of the contract—indicating that it is not optimal to exercise the contract on March 15, 2001.

Table 12.1 shows that in addition to relatively short-dated options, the exchanges trade longer-maturity contracts known as LEAPS, which were mentioned in Chapter 7. The acronym LEAPS stands for Long-term Equity AnticiPation Securities and was originated by the CBOE. LEAPS are exchange-traded options that last up to three years. The index is divided by five for the purposes of quoting the strike price and the option price. One contract is an option on 100 times one-fifth of the index (or 20 times the index). LEAPS on indices have expiration dates in December. As mentioned in Chapter 7, the CBOE and several other exchanges also trade LEAPS on many individual stocks. These have expirations in January.

The CBOE also trades *flex options* on indices. As mentioned in Chapter 7, these are options where the trader can choose the expiration date, the strike price, and whether the option is American or European.

Portfolio Insurance

Portfolio managers can use index options to limit their downside risk. Suppose that the value of an index today is S_0. Consider a manager in charge of a well-diversified portfolio whose beta is 1.0. A beta of 1.0 implies that the returns from the portfolio mirror those from the index. If the dividend yield from the portfolio is the same as the dividend yield from the index, the percentage changes in the value of the portfolio can be expected to be approximately the same as the percentage changes in the value of the index. Each contract on the S&P 500 is on 100 times the index. It follows that the value of the portfolio is protected against the possibility of the index falling below X if, for each $100S_0$ dollars in the portfolio, the manager buys one put option contract with strike price X. Suppose that the manager's portfolio is worth \$500,000 and the value of the index is 1,000. The portfolio is worth 500 times the index. The manager can obtain insurance against the value of the portfolio dropping below \$450,000 in the next three months by buying five put option contracts with a strike price of 900. To illustrate how this works, consider the situation where the index drops to 880 in three months. The portfolio will be worth about \$440,000. The payoff from the options will be $5 \times (900 - 880) \times 100 = \$10{,}000$, bringing the total value of the portfolio up to the insured value of \$450,000. This example is summarized in Table 12.2.

Table 12.2 Using options to protect the value of a portfolio that mirrors the S&P 500

From the Trader's Desk

A manager in charge of a portfolio worth $500,000 is concerned that the market might decline rapidly during the next three months and would like to use index options as a hedge. The portfolio is expected to mirror closely the S&P 500, which is currently standing at 1,000.

The Strategy

The manager buys five put option contracts with a strike price of 900. This strategy is designed to ensure that the value of the manager's position does not decline below $450,000.

The Outcome

The index dropped to 880 in the three months. The portfolio was worth $440,000. The payoff from the options was $5 \times (900 - 880) \times 100 = \$10{,}000$, bringing the total value of the position up to $\$440{,}000 + \$10{,}000 = \$450{,}000$.

When the Portfolio's Beta Is Not 1.0

If the portfolio's returns are not expected to equal those of an index, the capital asset pricing model can be used. This model asserts that the expected excess return of a portfolio over the risk-free interest rate equals beta times the excess return of a market index over the risk-free interest rate. Suppose that the $500,000 portfolio just considered has a beta of 2.0 instead of 1.0. Suppose further that the current risk-free interest rate is 12% per annum, the dividend yield on both the portfolio, and the index is expected to be 4% per annum. As before we assume that the S&P 500 index is currently 1,000. Table 12.3 shows the expected relationship between the level of the index and the value of the portfolio in three months. To illustrate the sequence of calculations necessary to derive Table 12.3, Table 12.4 shows what happens when the value of the index in three months proves to be 1,040.

Suppose that S_0 is the value of the index. It can be shown that for each $100S_0$ dollars in the portfolio, a total of beta put contracts should be purchased. The strike price should be the value that the index is expected to have when the value of the portfolio reaches the insured value. Suppose that the insured value is $450,000, as in the beta = 1.0 case. Table 12.3 shows that the appropriate strike price for the put options purchased is 960. In

Table 12.3 Relationship between value of index and value of portfolio for beta = 2.0

Value of index in three months	Value of portfolio in three months ($)
1,080	570,000
1,040	530,000
1,000	490,000
960	450,000
920	410,000
880	370,000

Table 12.4 Calculations for Table 12.3 when the value of the index is 1,040 in three months

Value of index in three months	1,040
Return from change in index	40/1,000, or 4% per three months
Dividends from index	$0.25 \times 4 = 1\%$ per three months
Total return from index	$4 + 1 = 5\%$ per three months
Risk-free interest rate	$0.25 \times 12 = 3\%$ per three months
Excess return from index over risk-free interest rate	$5 - 3 = 2\%$ per three months
Excess return from portfolio over risk-free interest rate	$2 \times 2 = 4\%$ per three months
Return from portfolio	$3 + 4 = 7\%$ per three months
Dividends from portfolio	$0.25 \times 4 = 1\%$ per three months
Increase in value of portfolio	$7 - 1 = 6\%$ per three months
Value of portfolio	\$500,000 × 1.06 = \$530,000

this case $100S_0 = \$100{,}000$ and beta $= 2.0$ so that two put contracts are required for each \$100,000 in the portfolio. Because the portfolio is worth \$500,000, a total of 10 contracts should be purchased.

To illustrate that the required result is obtained, consider what happens if the value of the index falls to 880. As shown in Table 12.3, the value of the portfolio is then about \$370,000. The put options pay off $(960 - 880) \times 10 \times 100 = \$80{,}000$, and this is exactly what is necessary to move the total value of the portfolio manager's position up from \$370,000 to the required level of \$450,000. This example is summarized in Table 12.5.

Valuation

In valuing index futures in Chapter 3, we assumed that the index could be treated as a security paying a known dividend yield. In valuing index options, we make similar assumptions. This means that equations (12.1) and (12.2) provide a lower bound for European index options; equation (12.3) is the put–call parity result for European index options; and equations (12.4) and (12.5) can be used to value European options on an index. In all cases S_0 is equal to the value of the index, σ is equal to the volatility of the index, and q is equal to the average annualized dividend yield on the index during the life of the option. The calculation of q should include only dividends whose ex-dividend date occurs during the life of the option.

In the United States ex-dividend dates tend to occur during the first week of February, May, August, and November. At any given time the correct value of q is therefore likely to depend on the life of the option. This is even more true for some foreign indices. For example, in Japan all companies tend to use the same ex-dividend dates.

Example

Consider a European call option on the S&P 500 that is two months from maturity. The current value of the index is 930, the exercise price is 900, the risk-free interest rate is 8% per annum, and the volatility of the index is 20% per annum. Dividend

Table 12.5 Using options to protect the value of a portfolio that has a beta of 2.0

From the Trader's Desk

A manager in charge of a portfolio worth \$500,000 is concerned that the market might decline rapidly during the next three months and would like to use index options as a hedge. The portfolio has a beta of 2.0 and the S&P 500 is standing at 1000. The dividend yield on both the index and the portfolio is expected to be 4% per annum, and the risk-free interest rate is 12% per annum.

The Strategy

The manager buys 10 put option contracts with a strike price of 960. The strategy is designed to ensure that the value of the manager's position does not decline below \$450,000.

The Outcome

The index dropped to 880 in the three months. The portfolio was worth \$370,000. The payoff from the options was $10 \times (960 - 880) \times 100 = \$80{,}000$, bringing the total value of the position up to $\$370{,}000 + \$80{,}000 = \$450{,}000$.

yields of 0.2% and 0.3% are expected in the first month and the second month, respectively. In this case $S_0 = 930$, $X = 900$, $r = 0.08$, $\sigma = 0.2$, and $T = 2/12$. The total dividend yield during the option's life is $0.2 + 0.3 = 0.5\%$. This is 3% per annum. Hence, $q = 0.03$ and

$$d_1 = \frac{\ln(930/900) + (0.08 - 0.03 + 0.2^2/2) \times 2/12}{0.2\sqrt{2/12}} = 0.5444$$

$$d_2 = \frac{\ln(930/900) + (0.08 - 0.03 - 0.2^2/2) \times 2/12}{0.2\sqrt{2/12}} = 0.4628$$

$$N(d_1) = 0.7069, \qquad N(d_2) = 0.6782$$

so that the call price, c, is given by equation (12.4) as

$$c = 930 \times 0.7069e^{-0.03\times2/12} - 900 \times 0.6782e^{-0.08\times2/12} = 51.83$$

One contract would cost \$5,183.

If the absolute amount of the dividend that will be paid on the stocks underlying the index (rather than the dividend yield) is assumed to be known, the basic Black–Scholes formula can be used with the initial stock price being reduced by the present value of the dividends. This is the approach recommended in Chapter 11 for a stock paying known dividends. However, the approach may be difficult to implement for a broadly based stock index because it requires a knowledge of the dividends expected on every stock underlying the index.

In some circumstances it is optimal to exercise American put options on an index prior to the exercise date. To a lesser extent, this is also true of American call options on an index. American stock index option prices are therefore always slightly more than the corresponding European stock index option prices. We will look at numerical procedures for valuing American index options in Chapter 17.

12.5 CURRENCY OPTIONS

European and American options on foreign currencies are actively traded in both the over-the-counter and exchange-traded market. The Philadelphia Stock Exchange commenced trading in currency options in 1982. Since then the size of the market has grown very rapidly. The currencies traded include the Australian dollar, British pound, Canadian dollar, German mark, Japanese yen, French franc, and Swiss franc. For most of these currencies, the Philadelphia Stock Exchange trades European as well as American options. A significant amount of trading in foreign currency options is also done outside organized exchanges. Many banks and other financial institutions are prepared to sell or buy foreign currency options that have strike prices, exercise dates, and other features tailored to meet the needs of their corporate clients.

For a corporation wishing to hedge a foreign exchange exposure, foreign currency options are an interesting alternative to forward contracts. A company due to receive sterling at a known time in the future can hedge its risk by buying put options on sterling that mature at that time. The strategy guarantees that the value of the sterling will not be less than the exercise price while allowing the company to benefit from any favorable exchange-rate movements. Similarly, a company due to pay sterling at a known time in the future can hedge by buying calls on sterling that mature at that time. The approach guarantees that the cost of the sterling will not be greater than a certain amount while allowing the company to benefit from favorable exchange-rate movements. Whereas a forward contract locks in the exchange rate for a future transaction, an option provides a type of insurance. This insurance is not free. It costs nothing to enter into a forward transaction, whereas options require a premium to be paid up front.

Quotes

Table 12.6 shows the closing prices of some of the currency options traded on the Philadelphia Stock Exchange on Thursday, March 15, 2001, as reported in the *Wall Street Journal* of Friday, March 16, 2001. The precise expiration date of a foreign currency option is the Saturday preceding the third Wednesday of the maturity month. The sizes of contracts are indicated at the beginning of each section of the table. The option prices are for the purchase or sale of one unit of a foreign currency with U.S. dollars. For the Japanese yen, the prices are in hundredths of a cent. For the other currencies, they are in cents. Thus, one call option contract on the euro with exercise price 90 cents and exercise month June would give the holder the right to buy 62,500 euros for U.S. \$56,250 ($= 0.90 \times 62{,}500$). The indicated price of the contract is 2.34 cents so that one contract would cost $62{,}500 \times 0.0234 = \$1{,}462.50$. The spot exchange rate is shown as 88.15 cents per pound sterling.

Valuation

To value currency options, we define S_0 as the spot exchange rate. To be precise, S_0 is the value of one unit of the foreign currency in U.S. dollars. As noted in Chapter 3, a foreign currency is analogous to a stock paying a known dividend yield. The owner of foreign currency receives a yield equal to the risk-free interest rate, r_f, in the foreign currency. Equations (12.1) and (12.2), with q replaced by r_f, provide bounds for the

Table 12.6 Currency option prices on the Philadelphia Exchange from the *Wall Street Journal* on March 16, 2001

PHILADELPHIA EXCHANGE OPTIONS

		CALLS VOL.	CALLS LAST	PUT VOL.	PUT LAST
ADllr					54.26
50,000 Australian Dollar EOM-European.					
51	Apr	4	0.44	...	...
ADollr					54.26
50,000 Australian Dollars-European Style.					
52	Mar	...	...	4	2.71
ADollr					54.26
50,000 Australian Dollars-cents per unit					
53	Jun	1	0.41	...	...
CDollr					66.48
50,000 Canadian Dollars-European Style.					
66	Mar	...	...	20	1.76
67^{50}	Mar	...	...	20	3.35
67^{50}	Sep	...	...	20	3.37
CDollr					66.48
50,000 Canadian Dollars-cents per unit.					
71^{50}	Mar	...	...	20	7.90
72^{50}	Mar	...	...	20	8.40
74^{50}	Mar	...	...	20	10.42
75	Mar	...	...	20	10.86
CDollr					66.48
50,000 Canadian Dollars-cents per unit.					
64	Jun	8	0.85	...	...
JYen					93.49
6,250,000 J.Yen EOM-European style.					
87^{50}	Mar	...	...	30	5.55
JYen					93.15
6,250,000J.Yen-100ths of a cent per unit.					
81^{50}	Mar	...	...	5	0.30
82^{50}	Apr	2	1.10	2	1.44
83	Apr	...	...	5	1.51
JYen					92.53
6,250,000J.Yen-EuropeanStyle.					
77	Mar	110	5.80	...	...
79	Sep	110	6.40	...	...
82	Mar	22	0.93	...	...
84	Jun	22	1.95	...	...
Euro					88.15
62,500 Euro-European style					
88	Mar	...	...	8	0.25
94	Jun	...	...	10	4.40
98	Jun	...	...	5	7.48
104	Mar	8	0.55	...	...
Euro					88.15
62,500 Euro-European style.					
92	Apr	2	0.73	...	...
96	Mar	...	...	5	5.38
Euro					88.15
62,500 Euro-cents per unit.					
88	Apr	...	...	10	0.43
90	Jun	22	2.34	1	2.35
94	Mar	...	...	3	3.65
114	Mar	...	...	2	23.55
SFranc					57.83
62,500 Swiss Franc EOM-European style.					
59	Apr	2	1.00	...	...
SFranc					57.83
62,500 Swiss Francs-European Style.					
58	Apr	16	1.30	...	...
60	Apr	16	0.34	...	...
SFranc					57.83
62,500 Swiss Francs-cents per unit.					
58	Apr	6	1.38	...	...
60	Sep	2	1.90	...	...
Call Vol. 1,221			Open Int. 12,520		
Put Vol. 3,926			Open Int. 12,955		

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European call price, c, and the European put price, p:

$$c \geqslant S_0 e^{-r_f T} - X e^{-rT}$$

$$p \geqslant X e^{-rT} - S_0 e^{-r_f T}$$

Equation (12.3), with q replaced by r_f, provides the put–call parity result for currency options:

$$c + X e^{-rT} = p + S_0 e^{-r_f T}$$

Finally, equations (12.4) and (12.5) provide the pricing formulas for currency options when q is replaced by r_f:

$$c = S_0 e^{-r_f T} N(d_1) - X e^{-rT} N(d_2) \qquad \textbf{(12.9)}$$

$$p = X e^{-rT} N(-d_2) - S_0 e^{-r_f T} N(-d_1) \qquad \textbf{(12.10)}$$

where

$$d_1 = \frac{\ln(S_0/X) + (r - r_f + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/X) + (r - r_f - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Both the domestic interest rate, r, and the foreign interest rate, r_f, are the rates for a maturity T. Put and call options on a currency are symmetrical in that a put option to sell currency A for currency B at an exercise price X is the same as a call option to buy B with A at $1/X$.

Example

Consider a four-month European call option on the British pound. Suppose that the current exchange rate is 1.6000, the exercise price is 1.6000, the risk-free interest rate in the United States is 8% per annum, the risk-free interest rate in Britain is 11% per annum, and the option price is 4.3 cents. In this case $S_0 = 1.6$, $X = 1.6$, $r = 0.08$, $r_f = 0.11$, $T = 0.3333$, and $c = 0.043$. The implied volatility can be calculated by trial and error. A volatility of 20% gives an option price of 0.0639; a volatility of 10% gives an option price of 0.0285; and so on. The implied volatility is 14.1%.

From equation (3.13), the forward rate, F_0, for a maturity T is given by

$$F_0 = S_0 e^{(r-r_f)T}$$

Thus, equations (12.9) and (12.10) can be simplified to

$$c = e^{-rT}[F_0 N(d_1) - XN(d_2)] \tag{12.11}$$

$$p = e^{-rT}[XN(-d_2) - F_0 N(-d_1)] \tag{12.12}$$

where

$$d_1 = \frac{\ln(F_0/X) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(F_0/X) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Note that for equations (12.11) and (12.12) to apply, the maturities of the forward contract and the option must be the same.

In some circumstances it is optimal to exercise American currency options prior to maturity. Thus, American currency options are worth more than their European counterparts. In general, call options on high-interest currencies and put options on low-interest currencies are the most likely to be exercised prior to maturity. The reason is that a high-interest currency is expected to depreciate relative to the U.S. dollar, and a low-interest currency is expected to appreciate relative to the U.S. dollar. Unfortunately, analytic formulas do not exist for the evaluation of American currency options. We will look at numerical procedures in Chapter 17.

12.6 SUMMARY

The Black–Scholes formula for valuing European options on a non-dividend-paying stock can be extended to cover European options on a stock paying a known dividend yield. This is useful because a number of other assets on which options are written can be considered to be analogous to a stock paying a dividend yield. This chapter has used the following results:

1. A stock index is analogous to a stock paying a dividend yield. The dividend yield is the dividend yield on the stocks comprising the index.
2. A foreign currency is analogous to a stock paying a dividend yield where the dividend yield is the foreign risk-free interest rate.

The extension to Black–Scholes can therefore be used to value European options on stock indices and foreign currencies. As we will see in Chapter 17, these analogies are also useful in numerically valuing American options on these assets.

Index options are settled in cash. On exercise of an index call option contract, the holder receives 100 times the amount by which the index exceeds the strike price. Similarly, on exercise of an index put option, the holder receives 100 times the amount by which the strike price exceeds the index. Index options can be used for portfolio insurance. If the value of the portfolio mirrors the index, it is appropriate to buy one put option contract for each $100S_0$ dollars in the portfolio, where S_0 is the value of the index. If the portfolio does not mirror the index, beta put option contracts should be purchased for each $100S_0$ dollars in the portfolio, where beta is the beta of the portfolio calculated using the capital asset pricing model. The strike price of the put options purchased should reflect the level of insurance required.

Currency options are traded both on organized exchanges and over the counter. They can be used by corporate treasurers to hedge foreign exchange exposure. For example, a U.S. corporate treasurer who knows that the company will be receiving sterling at a certain time in the future can hedge by buying put options that mature at that time. Similarly, a U.S. corporate treasurer who knows sterling will be paid at a certain time in the future can hedge by buying call options that mature at that time.

Suggestions for Further Reading

General

Merton, R. C. "Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science* 4 (spring 1973): 141–83.

Stoll, H. R., and R. E. Whaley. "New Option Instruments; Arbitrageable Linkages and Valuation." *Advances in Futures and Options Research* 1, pt. A (1986): 25–62.

On options on stock indices

Chance, D. M. "Empirical Tests of the Pricing of Index Call Options." *Advances in Futures and Options Research* 1, pt. A (1986): 141–66.

On options on currencies

Amin, K., and R. A. Jarrow. "Pricing Foreign Currency Options under Stochastic Interest Rates." *Journal of International Money and Finance* 10 (1991): 310–29.

Biger, N., and J. C. Hull. "The Valuation of Currency Options." *Financial Management* 12 (spring 1983): 24–28.

Bodurtha, J. N., and G. R. Courtadon. "Tests of an American Option Pricing Model on the Foreign Currency Options Market." *Journal of Financial and Quantitative Analysis* 22 (June 1987): 153–67.

Garman, M. B., and S. W. Kohlhagen. "Foreign Currency Option Values." *Journal of International Money and Finance* 2 (December 1983): 231–37.

Grabbe, J. O. "The Pricing of Call and Put Options on Foreign Exchange." *Journal of International Money and Finance* 2 (December 1983): 239–53.

Quiz (Answers at End of Book)

12.1. A portfolio is currently worth $10 million and has a beta of 1.0. The S&P 100 is currently standing at 800. Explain how a put option on the S&P 100 with a strike price of 700 can be used to provide portfolio insurance.

12.2. "Once we know how to value options on a stock paying a dividend yield, we know how to value options on stock indices and currencies." Explain this statement.

12.3. A stock index is currently 300, the dividend yield on the index is 3% per annum, and the risk-free interest rate is 8% per annum. What is a lower bound for the price of a six-month European call option on the index when the strike price is 290?

12.4. A currency is currently worth \$0.80. Over each of the next two months it is expected to increase or decrease in value by 2%. The domestic and foreign risk-free interest rates are 6% and 8%, respectively. What is the value of a two-month European call option with a strike price of \$0.80?

12.5. Explain how corporations can use currency options to hedge their foreign exchange risk.

12.6. Calculate the value of a three-month at-the-money European call option on a stock index when the index is at 250, the risk-free interest rate is 10% per annum, the volatility of the index is 18% per annum, and the dividend yield on the index is 3% per annum.

12.7. Calculate the value of an eight-month European put option on a currency with a strike price of 0.50. The current exchange rate is 0.52, the volatility of the exchange rate is 12%, the domestic risk-free interest rate is 4% per annum, and the foreign risk-free interest rate is 8% per annum.

Questions and Problems (Answers in Solutions Manual)

12.8. Suppose that an exchange constructs a stock index that tracks the return, including dividends, on a certain portfolio. Explain how you would value (a) futures contracts and (b) European options on the index.

12.9. A foreign currency is currently worth \$1.50. The domestic and foreign risk-free interest rates are 5% and 9%, respectively. Calculate a lower bound for the value of a six-month call option on the currency with a strike price of \$1.40 if it is (a) European and (b) American.

12.10. Consider a stock index currently standing at 250. The dividend yield on the index is 4% per annum, and the risk-free rate is 6% per annum. A three-month European call option on the index with a strike price of 245 is currently worth \$10. What is the value of a three-month put option on the index with a strike price of 245?

12.11. An index currently stands at 696 and has a volatility of 30% per annum. The risk-free rate of interest is 7% per annum and the index provides a dividend yield of 4% per annum. Calculate the value of a three-month European put with an exercise price of 700.

12.12. Show that if C is the price of an American call with exercise price X and maturity T on a stock paying a dividend yield of q, and P is the price of an American put on the same stock with the same strike price and exercise date,

$$S_0e^{-qT} - X < C - P < S_0 - Xe^{-rT}$$

where S_0 is the stock price, r is the risk-free rate, and $r > 0$. (*Hint*: To obtain the first half of the inequality, consider possible values of:

Portfolio A: a European call option plus an amount X invested at the risk-free rate

Portfolio B: an American put option plus e^{-qT} of stock with dividends being reinvested in the stock

To obtain the second half of the inequality, consider possible values of:

Portfolio C: an American call option plus an amount Xe^{-rT} invested at the risk-free rate

Portfolio D: a European put option plus one stock with dividends being reinvested in the stock)

12.13. Show that a call option on a currency has the same price as the corresponding put option on the currency when the forward price equals the strike price.

12.14. Would you expect the volatility of a stock index to be greater or less than the volatility of a typical stock? Explain your answer.

12.15. Does the cost of portfolio insurance increase or decrease as the beta of a portfolio increases? Explain your answer.

12.16. Suppose that a portfolio is worth $60 million and the S&P 500 is at 1200. If the value of the portfolio mirrors the value of the index, what options should be purchased to provide protection against the value of the portfolio falling below $54 million in one year's time?

12.17. Consider again the situation in Problem 12.16. Suppose that the portfolio has a beta of 2.0, the risk-free interest rate is 5% per annum, and the dividend yield on both the portfolio and the index is 3% per annum. What options should be purchased to provide protection against the value of the portfolio falling below $54 million in one year's time?

Assignment Questions

12.18. Use the DerivaGem software to calculate implied volatilities for the June 100 call and the June 100 put on the Dow Jones Industrial Average in Table 12.1. The value of the DJX on March 15, 2001, was 100.31. Assume the risk-free rate was 4.5%, the dividend yield was 2%. The options expire on June 16, 2001. Are the quotes for the two options consistent with put–call parity?

12.19. A stock index currently stands at 300. It is expected to increase or decrease by 10% over each of the next two time periods of three months. The risk-free interest rate is 8% and the dividend yield on the index is 3%. What is the value of a six-month put option on the index with a strike price of 300 if it is (a) European and (b) American?

12.20. Suppose that the spot price of the Canadian dollar is U.S. $0.75 and that the Canadian dollar/U.S. dollar exchange rate has a volatility of 4% per annum. The risk-free rates of interest in Canada and the United States are 9% and 7% per annum, respectively. Calculate the value of a European call option to buy one Canadian dollar for U.S. $0.75 in nine months. Use put–call parity to calculate the price of a European put option to sell one Canadian dollar for U.S. $0.75 in nine months. What is the price of a call option to buy U.S. $0.75 with one Canadian dollar in nine months?

12.21. A mutual fund announces that the salaries of its fund managers will depend on the performance of the fund. If the fund loses money, the salaries will be zero. If the fund makes a profit, the salaries will be proportional to the profit. Describe the salary of a fund manager as an option. How is a fund manager motivated to behave with this type of remuneration package?