CHAPTER 13

Futures Options

The options we have considered so far provide the holder with the right to buy or sell a certain asset by a certain date. They are sometimes termed *options on spot* or *spot options* because, when the options are exercised, the sale or purchase of the asset at the agreed-on price takes place immediately. In this chapter we move on to consider *options on futures*, also known as *futures options*. In these contracts, exercise of the option gives the holder a position in a futures contract.

The Commodity Futures Trading Commission authorized the trading of options on futures on an experimental basis in 1982. Permanent trading was approved in 1987, and since then the popularity of the contract with investors has grown very fast.

In this chapter we consider how futures options work and the differences between futures options and spot options. We examine how European futures options can be priced using either binomial trees or formulas similar to the ones produced by Black and Scholes for stock options. We also explore the relative pricing of futures options and spot options.

13.1 NATURE OF FUTURES OPTIONS

A futures option is the right, but not the obligation, to enter into a futures contract at a certain futures price by a certain date. Specifically, a call futures option is the right to enter into a long futures contract at a certain price; a put futures option is the right to enter into a short futures contract at a certain price. Most futures options are American; that is, they can be exercised any time during the life of the contract.

To illustrate the operation of futures options contracts, consider the position of an investor who has bought a July call futures option on gold with a strike price of $300 per ounce. The asset underlying one contract is 100 ounces of gold. As with other exchange-traded option contracts, the investor is required to pay for the option at the time the contract is entered into. If the call futures option is exercised, the investor obtains a long futures contract, and there is a cash settlement to reflect the investor entering into the futures contract at the strike price. Suppose that the July futures price at the time the option is exercised is 340 and the most recent settlement price for the July futures

Table 13.1 Call futures options

From the Trader's Desk

An investor buys a July call futures option contract on gold. The contract size is 100 ounces. The strike price is 300.

The Exercise Decision

The investor exercises when the July gold futures price is 340 and the most recent settlement price is 338.

The Outcome

The investor receives a long futures contract plus a cash amount equal to (338 − 300) × 100 = \$3,800. The investor decides to close out the long futures position immediately for a gain of (340 − 338) × 100 = \$200. The total payoff from the decision to exercise is therefore \$4,000.

contract is 338. The investor receives a cash amount equal to the excess of the most recent settlement price over the strike price. This amount, (338 − 300) × 100 = \$3,800 in our example, is added to the investor's margin account.

If the investor closes out the July futures contract immediately, the gain on the futures contract is (340 − 338) × 100, or \$200. The total payoff from exercising the futures option contract is therefore \$4,000. This corresponds to the July futures price at the time of exercise less the strike price. If the investor keeps the futures contract, additional margin may be required. The example is summarized in Table 13.1.

The investor who sells (or writes) a call futures option receives the option premium, but takes the risk that the contract will be exercised. When the contract is exercised, this investor assumes a short futures position. An amount equal to $F - X$ is deducted from the investor's margin account, where F is the most recent settlement price. The exchange clearinghouse arranges for this sum to be transferred to the investor on the other side of the transaction who chose to exercise the option.

Put futures options work analogously to call options. Consider an investor who buys a September put futures option on corn with a strike price of 200 cents per bushel. Each contract is on 5,000 bushels of corn. If the put futures option is exercised, the investor obtains a short futures contract plus a cash settlement. Suppose the contract is exercised when the September futures price is 180 cents and the most recent settlement price is 179 cents. The investor receives a cash amount equal to the excess of the strike price over the most recent settlement price. The cash amount received, which is (2.00 − 1.79) × 5,000 = \$1,050 in our example, is added to the investor's margin account. If the investor closes out the futures contract immediately, the loss on the short futures contract is (1.80 − 1.79) × 5,000 = \$50. The total payoff from exercising the futures option contract is therefore \$1,000. This corresponds to the strike price minus the futures price at the time of exercise. As in the case of call futures, additional margin may be required if the investor decides to keep the futures position. The example is summarized in Table 13.2.

The investor on the other side of the transaction (i.e., the investor who sold the put futures option) obtains a long futures position when the option is exercised, and the excess of the strike price over the most recent settlement price is deducted from the investor's margin account.

Table 13.2 Put futures options

From the Trader's Desk

An investor buys a September put futures option contract on corn. The contract size is 5,000 bushels. The strike price is 200 cents.

The Exercise Decision

The investor exercises when the September corn futures price is 180 and the most recent settlement price is 179.

The Outcome

The investor receives a short futures contract plus a cash amount of $(2.00 - 1.79) \times 5{,}000 = \$1{,}050$. The investor decides to close out the short futures position immediately for a loss of $(1.80 - 1.79) \times 5{,}000 = \$50$. The total payoff from the decision to exercise is therefore $1,000.

13.2 QUOTES

As mentioned earlier, most futures options are American. They are referred to by the month in which the underlying futures contract matures—not by the expiration month of the option. The maturity date of a futures option contract is usually on, or a few days before, the earliest delivery date of the underlying futures contract. For example, the S&P 500 index futures options expire on the same day as the underlying futures contract; the CME currency futures options expire two business days prior to the expiration of the futures contract; the CBOT Treasury bond futures option expires on the first Friday preceding by at least five business days the end of the month, just prior to the futures contract expiration month. An exception is the CME mid-curve Eurodollar contract where the futures contract expires either one or two years after the options contract.

Table 13.3 shows quotes for futures options as they appeared in the *Wall Street Journal* on March 16, 2001. The most popular contracts include those on corn, soybeans, wheat, sugar, crude oil, heating oil, natural gas, gold, Treasury bonds, Treasury notes, five-year Treasury notes, Eurodollars, one-year mid-curve Eurodollars, Euribor, Eurobunds, and the S&P 500.

13.3 REASONS FOR THE POPULARITY OF FUTURES OPTIONS

It is natural to ask why people choose to trade options on futures rather than options on the underlying asset. The main reason appears to be that a futures contract is, in many circumstances, more liquid and easier to trade than the underlying asset. Furthermore, a futures price is known immediately from trading on the futures exchange, whereas the spot price of the underlying asset may not be so readily available.

Consider Treasury bonds. The market for Treasury bond futures is much more active than the market for any particular Treasury bond. Also, a Treasury bond futures price is known immediately from trading on the Chicago Board of Trade. By contrast, the current market price of a bond can be obtained only by contacting one or more dealers.

Table 13.3 Closing prices of futures options on March 15, 2001

FUTURES OPTIONS PRICES

Thursday, March 15, 2001

AGRICULTURAL

Corn (CBT)
5,000 bu.; cents per bu.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jly	Apr	May	Jly
190	....	20¾	....	....	⅛	1
200	....	12	....	⅛	1¼	2¾
210	....	5¼	14½	2	4¾	6
220	½	2¼	9⅝	9¾	11½	11⅛
230	⅛	⅞	6⅜	19¼	20	17¾
240	⅛	½	4½	....	29½	25½

Est vol 25,000 Wd 10,993 calls 6,347 puts
Op int Wed 265,371 calls 147,975 puts

Soybeans (CBT)
5,000 bu.; cents per bu.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jly	Apr	May	Jly
400	....	46	53¼	....	½	2¼
420	....	28	36⅛	¼	2¼	5½
440	8½	13¾	23⅞	3	8½	12¾
460	1⅛	5¾	15¾	15¾	20	24
480	⅛	2¼	10½	34½	36½	38¼
500	⅛	¾	6¾	54½	55	54¾

Est vol 15,000 Wd 16,691 calls 6,884 puts
Op int Wed 122,249 calls 53,376 puts

Soybean Meal (CBT)
100 tons; $ per ton

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jly	Apr	May	Jly
140	....	....	....	0.10	0.85	....
145	....	....	....	0.25	1.50	3.75
150	....	5.00	....	....	3.25	5.75
155	1.00	2.75	4.75	4.00	5.85	8.75
160	0.10	1.50	3.35	8.25	9.60	12.40
165	....	0.90	2.35	....	13.90	16.40

Est vol 2,500 Wd 2,266 calls 3,582 puts
Op int Wed 21,870 calls 18,231 puts

Soybean Oil (CBT)
60,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jly	Apr	May	Jly
150	1.100	1.150	1.590	.030	.090	.160
155	....	.750	1.200	.060	.190	.280
160	.250	.450	.920	.200	.400	.500
165	.100	.260	.700	.520	.700	.770
170	.030	.150	.550	....	1.090	1.120
175	.005	.090	.400	....	....	....

Est vol 1,400 Wd 598 calls 350 puts
Op int Wed 40,326 calls 16,503 puts

Wheat (CBT)
5,000 bu.; cents per bu.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jly	Apr	May	Jly
250	....	24¼	36½	⅛	½	2
260	....	16	28¾	¼	2¼	4
270	5	9½	22	1¼	5¾	7¼
280	1½	5½	16½	7¾	11¾	11½
290	½	2⅞	12	....	19	17
300	⅛	1⅝	9	....	28	24

Est vol 12,000 Wd 1,397 calls 1,133 puts
Op int Wed 72,666 calls 50,935 puts

Cotton (NYBOT)
50,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jly	Oct	May	Jly	Oct
48	....	4.25	....	0.80	1.00	....
49	....	....	....	1.10	1.33	....
50	1.70	3.01	....	1.61	1.73	....
51	1.26	....	....	2.17	2.18	....
52	0.93	2.02	....	2.83	2.71	....
53	0.68	1.71	....	3.58	3.38	....

Est vol 8,300 Wd 3,159 calls 1,262 puts
Op int Wed 73,892 calls 39,850 puts

Orange Juice (NYBOT)
15,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jly	Sep	May	Jly	Sep
60	....	....	....	....	....	....
65	....	....	....	0.10	....	....
70	5.05	8.80	....	0.35	....	0.65
75	1.55	5.15	8.20	1.75	1.85	1.65
80	0.35	2.65	5.60	5.65	4.15	3.50
85	0.20	1.30	....	10.45	7.70	6.50

Est vol 300 Wd 124 calls 30 puts
Op int Wed 14,097 calls 14,686 puts

Coffee (NYBOT)
37,500 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jun	Jly	May	Jun	Jly
55	6.33	9.46	9.88	0.35	0.55	1.00
57.5	4.44	7.47	8.10	0.95	1.05	1.70
60	3.00	5.88	6.57	1.90	1.95	2.65
62.5	1.90	4.44	5.29	3.40	3.00	3.85
65	1.20	3.40	4.25	5.19	4.44	5.29
67.5	0.75	2.50	3.40	7.23	6.03	6.92

Est vol 3,176 Wd 1,898 calls 1,213 puts
Op int Wed 44,460 calls 14,659 puts

Sugar-World (NYBOT)
112,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jun	Jly	May	Jun	Jly
800	0.98	0.70	0.81	0.06	0.28	0.39
850	0.58	0.45	0.56	0.16	0.53	0.64
900	0.31	0.27	0.38	0.39	0.85	0.96
950	0.14	0.16	0.25	0.72	1.23	1.32
1000	0.06	0.09	0.16	1.14	1.66	1.73
1050	0.02	0.05	0.11	1.60	2.12	2.18

Est vol 5,293 Wd 5,532 calls 1,636 puts
Op int Wed 68,646 calls 52,440 puts

Cocoa (NYBOT)
10 metric tons; $ per ton

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jun	Jly	May	Jun	Jly
900	117	135	142	2	8	15
950	75	97	107	10	19	30
1000	42	67	81	27	39	53
1050	20	42	57	55	64	79
1100	10	26	42	93	98	113
1150	7	16	31	142	137	152

Est vol 2,376 Wd 369 calls 450 puts
Op int Wed 25,586 calls 18,254 puts

OIL

Crude Oil (NYM)
1,000 bbls.; $ per bbl.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
2550	1.05	1.97	2.44	0.01	0.66	0.98
2600	0.55	1.66	2.13	0.01	0.84	1.17
2650	0.05	1.37	1.85	0.01	1.05	1.38
2700	0.01	1.11	1.57	0.45	1.29	1.60
2750	0.01	0.90	1.35	0.95	1.58	1.88
2800	0.01	0.69	1.13	1.45	1.87	2.15

Est vol 50,195 Wd 15,823 calls 23,794 puts
Op int Wed 298,568 calls 366,294 puts

Heating Oil No.2 (NYM)
42,000 gal.; $ per gal.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
69	.0295	.0318	.0391	.0130	.0331	.0419
70	.0230	.0277	.0347	.0165	.0390	.0475
71	.0196	.0240	.0307	.0231	.0452	.0534
72	.0140	.0207	.0271	.0275	.0519	.0597
73	.0100	.0179	.0238	.0335	.0590	.0663
74	.0085	.0154	.0208	.0419	.0664	.0732

Est vol 4,562 Wd 1,672 calls 1,373 puts
Op int Wed 37,794 calls 25,749 puts

Gasoline-Unlead (NYM)
42,000 gal.; $ per gal.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
85	.0339	.0474	.0504	.0160	.0360	.0495
86	.0282	.0424	.0462	.0203	.0410	.0553
87	.0231	.0380	.0422	.0252	.0466	.0612
88	.0189	.0340	.0386	.0310	.0525	.0675
89	.0153	.0304	.0352	.0374	.0589	.0740
90	.0110	.0270	.0321	.0430	.0654	.0808

Est vol 3,077 Wd 1,768 calls 676 puts
Op int Wed 77,653 calls 23,889 puts

Natural Gas (NYM)
10,000 MMBtu.; $ per MMBtu.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
485	.202	....	.476	.125	....	.327
490	.174	.331	.451	.147	.271	.351
495	.149	.305	....	.172	.295	....
500	.130	.283	.400	.203	.323	.400
505	.112	.258	.377	.235	.348	....
510	.096	.238	.354	.269	.378	.454

Est vol 15,918 Wd na calls na puts
Op int Wed 188,492 calls 212,884 puts

Brent Crude (IPE)
1,000 net bbls.; $ per bbl.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jun	Jly	May	Jun	Jly
2400	1.51	2.06	2.50	0.50	0.84	1.21
2450	1.20	1.78	2.20	0.69	1.06	1.41
2500	0.93	1.54	1.92	0.92	1.32	1.63
2550	0.71	1.32	1.66	1.20	1.60	1.87
2600	0.53	1.11	1.43	1.52	1.89	2.14
2650	0.39	0.92	1.23	1.88	2.20	2.44

Est vol 700 Wd 600 calls 0 puts
Op int Wed 8,139 calls 12,408 puts

Gas Oil (IPE)
100 metric tons; $ per ton

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
20000	12.75	14.65	18.05	3.50	5.65	7.80
20500	9.60	12.00	15.20	3.53	8.00	9.95
21000	7.00	9.50	12.65	7.75	10.50	12.40
21500	4.85	7.65	10.45	10.60	13.65	15.20
22000	3.15	6.15	8.55	13.90	17.15	18.30
22500	2.00	4.70	6.95	....	20.70	21.70

Est vol 175 Wd 1,810 calls 0 puts
Op int Wed 3,799 calls 467 puts

LIVESTOCK

Cattle-Feeder (CME)
50,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Mar	Apr	May	Mar	Apr	May
8450	....	....	....	....	....	....
8500	0.65	1.70	....	0.20	0.75	....
8550	....	....	....	....	....	....
8600	0.20	1.15	1.77	0.75	1.25	1.70
8650	0.12	....	....	....	....	....
8700	0.05	....	....	1.60	1.70	....

Est vol 628 Wd 376 calls 787 puts
Op int Wed 6,063 calls 20,057 puts

Cattle-Live (CME)
40,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	Jun	Aug	Apr	Jun	Aug
76	2.42	0.55	0.67	0.45	3.97	....
77	1.70	0.32	....	0.72	....	....
78	1.15	0.22	0.30	1.17	....	....
79	0.70	0.15	....	1.72	....	....
80	0.40	0.07	0.22	2.42	....	....
81	0.22	....	....	3.25	8.42	....

(continued on next page)

Table 13.3 *(continued)*

Est vol 3,375 Wd 1,688 calls 2,277 puts
Op int Wed 32,648 calls 63,310 puts

Hogs-Lean (CME)
40,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	Jun	Jly	Apr	Jun	Jly
64	3.47	7.92	5.67	1.80	1.10	2.50
65	2.87	7.20	....	2.20	1.35	....
66	2.32	6.55	4.45	2.65	1.70	3.25
67	1.85	5.97	....	....	2.10	....
68	1.50	5.37	3.45	....	2.50	4.20
69	1.20	4.80	....	....	2.90	....

Est vol 1,195 Wd 491 calls 568 puts
Op int Wed 7,287 calls 5,918 puts

METALS

Copper (CMX)
25,000 lbs.; cents per lb.

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
76	4.75	5.80	6.45	0.15	0.80	1.20
78	3.05	4.25	5.00	0.45	1.25	1.70
80	1.70	2.95	3.80	1.10	1.90	2.50
82	0.80	1.85	2.80	2.20	2.80	3.50
84	0.35	1.15	2.05	3.70	4.10	4.70
86	0.10	0.65	1.40	5.50	5.60	6.10

Est vol 125 Wd 26 calls 69 puts
Op int Wed 2,926 calls 942 puts

Gold (CMX)
100 troy ounces; $ per troy ounce

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jun	Aug	May	Jun	Aug
250	14.00	14.20	16.90	1.50	2.20	3.70
255	9.40	11.20	13.30	2.20	3.60	5.10
260	6.00	7.60	10.50	3.90	5.30	7.00
265	3.90	5.40	8.50	6.90	7.70	10.20
270	2.20	3.90	6.70	10.20	11.60	13.10
275	1.50	2.90	5.50	14.30	15.60	16.70

Est vol 13,000 Wd 5,232 calls 2,333 puts
Op int Wed 221,107 calls 68,332 puts

Silver (CMX)
5,000 troy ounces; cts per troy ounces

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	May	Jun	Jly	May	Jun	Jly
375	60.5	65.2	65.2	0.3	1.2	1.0
400	36.3	41.0	41.7	1.0	1.8	2.5
425	13.8	19.5	22.0	3.5	5.3	7.8
450	3.3	8.0	9.5	18.0	18.8	20.3
475	1.3	3.4	5.3	41.0	39.1	41.1
500	0.8	2.2	3.2	65.5	63.0	64.0

Est vol 1,500 Wd 1,708 calls 496 puts
Op int Wed 37,475 calls 12,284 puts

INTEREST RATE

T-Bonds (CBT)
$100,000; points – 64ths of 100%

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
104	2-11	2-35	2-61	0-03	0-28	0-54
105	1-20	1-54	2-20	0-11	0-47	1-13
106	0-41	1-18	1-49	0-32	1-10	1-41
107	0-16	0-54	1-19	1-06	1-46	....
108	0-05	0-34	0-60	1-61	....	2-52
109	0-01	0-19	0-44	....	....	3-33

Est vol 53,000;
Wd vol 42,385 calls 34,000 puts
Op int Wed 239,691 calls 181,312 puts

T-Notes (CBT)
$100,000; points – 64ths of 100%

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
105	1-34	1-53	2-10	0-04	0-23	0-44
106	0-46	1-11	1-33	0-16	0-45	1-04
107	0-16	0-45	1-04	0-50	....	1-38
108	0-05	0-25	0-46	....	....	2-16
109	0-02	0-14	0-30	2-35	....	2-63
110	0-01	....	0-19	....	....	3-51

Est vol 65,000 Wd 44,195 calls 41,055 puts
Op int Wed 346,907 calls 249,181 puts

5 Yr Treas Notes (CBT)
$100,000; points – 64ths of 100%

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
10450	1-11	2-13	1-37	0-03	0-16	0-30
10500	0-47	....	1-17	0-07	....	0-41
10550	0-26	1-23	0-63	0-18	0-38	0-55
10600	0-13	1-01	0-48	....	0-55	....
10650	0-05	0-46	0-36	....	....	....
10700	0-03	0-32	0-26	....	....	....

Est vol 23,000 Wd 12,507 calls 10,564 puts
Op int Wed 126,217 calls 97,191 puts

Eurodollar (CME)
$ million; pts of 100%

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Mar	Apr	May	Mar	Apr	May
9450	5.92	10.30	....	0.00	0.00	....
9475	3.42	7.80	....	0.00	0.00	0.07
9500	0.95	5.30	5.50	0.02	0.00	0.22
9525	0.05	3.15	3.35	1.62	0.35	....
9550	0.00	1.50	....	....	1.20	....
9575	0.00	0.65	....	....	....	....

Est vol 233,398;
Wd vol 183,411 calls 101,539 puts
Op int Wed 1,845,759 calls 1,911,991 puts

1 Yr. Mid-Curve Eurodlr (CME)
$1 million contract units; pts of 100%

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Mar	Apr	May	Mar	Apr	May
9475	6.20	4.25	4.65	0.00	0.30	....
9500	3.70	2.40	2.85	0.00	0.95	1.40
9525	1.30	1.10	1.85	0.10	....	....
9550	0.15	0.35	0.80	....	....	....
9575	0.00	0.10	....	....	....	....
9600	....	....	....	....	....	....

Est vol 72,910 Wd 60,260 calls 31,975 puts
Op int Wed 671,496 calls 460,417 puts

2 Yr. Mid-Curve Eurodlr (CME)
$1million contract units; pts of 100%

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Mar	Jun		Mar	Jun	
9425	4.10	4.35	....	0.00	1.25	....
9450	1.65	2.90	....	0.05	2.25	....
9475	....	1.75	....	....	....	....
9500	....	....	....	....	....	....
9525	....	....	....	....	....	....
9550	....	....	....	....	....	....

Est vol 250 Wd 0 calls 0 puts
Op int Wed 16,185 calls 19,908 puts

Euribor (LIFFE)
Euro 1,000,000

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Mar	Apr	May	Mar	Apr	May
95000	0.25	0.58	0.58	....	....	....
95125	0.12	0.46	0.46	....	....	0.00
95250	0.01	0.33	0.34	0.01	....	0.01
95375	....	0.22	0.24	0.12	0.01	0.03
95500	....	0.12	0.15	0.25	0.03	0.06
95625	....	0.05	0.09	0.37	0.09	0.13

Vol Th 48,655 calls 9,669 puts
Op int Wed 886,103 calls 752,794 puts

10 Yr. German Euro Gov't Bd (Eurobund) (Eurex)100,000;pts in 100%

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
10850	1.16	1.33	1.52	0.02	0.19	0.38
10900	0.72	0.98	1.19	0.08	0.33	0.55
10950	0.37	0.68	0.90	0.24	0.53	0.77
11000	0.15	0.45	0.67	0.52	0.81	1.03
11050	0.06	0.28	0.50	0.92	1.14	1.36
11100	0.01	0.17	0.36	1.37	1.52	1.72

Vol Th 55,671 calls 28,301 puts
Op int Wed 586,672 calls 353,718 puts

CURRENCY

Japanese Yen (CME)
12,500,000 yen; cents per 100 yen

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
8050	....	....	....	0.38	....	1.29
8100	....	....	3.12	0.51	0.98	1.44
8150	....	....	....	0.64	1.13	....
8200	1.54	....	....	0.84	1.32	1.80
8250	....	....	....	1.04	1.54	2.02
8300	0.95	....	1.96	1.25	1.78	2.26

Est vol 3,247 Wd 8,400 calls 9,234 puts
Op int Wed 39,710 calls 38,759 puts

Deutschemark (CME)
125,000 marks; cents per mark

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
4500	....	....	....	....	....	....
4550	....	....	....	....	....	....
4600	....	....	....	....	....	....
4650	....	....	....	....	....	....
4700	....	....	....	....	....	....
4750	....	....	....	....	....	....

Est vol 5 Wd 1 calls 1 puts
Op int Wed 440 calls 19 puts

Canadian Dollar (CME)
100,000 Can.$, cents per Can.$

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
6300	....	....	1.42	....	0.21	0.35
6350	....	....	....	0.17	0.33	0.51
6400	....	....	0.80	0.35	0.53	0.72
6450	0.22	0.40	....	0.64	....	0.98
6500	0.10	0.25	0.40	1.02	1.16	1.31
6550	0.05	....	0.28	1.47	....	1.68

Est vol 176 Wd 370 calls 376 puts
Op int Wed 15,899 calls 3,557 puts

British Pound (CME)
62,500 pounds; cents per pound

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
1420	....	....	3.48	0.62	....	1.76
1430	....	....	....	0.98	1.72	....
1440	1.24	....	2.46	1.50	....	2.72
1450	0.84	1.46	1.98	2.10	2.72	3.22
1460	0.52	1.08	1.58	2.78	3.22	3.82
1470	0.30	0.80	1.32	3.56	3.82	4.54

Est vol 258 Wd 68 calls 231 puts
Op int Wed 4,499 calls 3,507 puts

Swiss Franc (CME)
125,000 francs; cents per franc

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Apr	May	Jun	Apr	May	Jun
5750	1.51	....	....	....	....	....
5800	....	....	....	....	....	1.00
5850	....	....	....	0.52	....	....
5900	0.57	....	1.28	0.78	....	1.48
5950	0.39	....	....	1.10	....	....
6000	0.27	....	0.89	1.48	....	2.08

Est vol 410 Wd 79 calls 177 puts
Op int Wed 6,888 calls 3,808 puts

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DJ Industrial Avg (CBOT)
$100 times premium

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Mar	Apr	May	Mar	Apr	May
98	....	49.20	....	0.05	18.80	....
99	....	....	....	0.15	21.95	....
100	2.50	....	....	2.15	25.60	36.00
101	0.05	30.20	....	7.50	29.70	....
102	0.05	24.85	35.00	17.50	34.30	....
103	0.05	20.50	....	27.50	39.95	....

Est vol 1,100 Wd 974 calls 1,839 puts
Op int Wed 7,614 calls 11,906 puts

S&P 500 Stock Index (CME)
$250 times premium

STRIKE	CALLS-SETTLE			PUTS-SETTLE		
PRICE	Mar	Apr	May	Mar	Apr	May
1165	....	....	....	2.90	30.80	44.00
1170	7.80	47.30	....	4.50	32.70	46.00
1175	5.00	44.30	....	6.70	34.70	48.10
1180	3.00	41.50	....	9.70	36.80	50.20
1185	1.70	38.70	47.80	13.40	39.00	52.20
1190	1.10	36.00	45.40	17.80	41.30	54.50

Est vol 22,943 Wd 10,913 calls 32,911 puts
Op int Wed 111,149 calls 220,300 puts

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It is not surprising that investors would rather take delivery of a Treasury bond futures contract than Treasury bonds.

Futures on commodities are also often easier to trade than the commodities themselves. For example, it is much easier and more convenient to make or take delivery of a live-hogs futures contract than it is to make or take delivery of the hogs themselves.

An important point about a futures option is that exercising it does not usually lead to delivery of the underlying asset, as in most circumstances the underlying futures contract is closed out prior to delivery. Futures options are therefore normally eventually settled in cash. This is appealing to many investors, particularly those with limited capital who may find it difficult to come up with the funds to buy the underlying asset when an option is exercised.

Another advantage sometimes cited for futures options is that futures and futures options are traded in pits side by side in the same exchange. This facilitates hedging, arbitrage, and speculation. It also tends to make the markets more efficient.

A final point is that futures options tend to entail lower transactions costs than spot options in many situations.

13.4 PUT–CALL PARITY

In Chapter 8 we derived a put–call parity relationship for European stock options. We now consider a similar argument to derive a put–call parity relationship for European futures options.

Consider European call and put futures options, both with strike price X and time to expiration T. We can form two portfolios:

Portfolio A: a European call futures option plus an amount of cash equal to Xe^{-rT}

Portfolio B: a European put futures option plus a long futures contract plus an amount of cash equal to F_0e^{-rT}, where F_0 is the futures price

In portfolio A the cash can be invested at the risk-free rate, r, and grows to X at time T. Let F_T be the futures price at maturity of the option. If $F_T > X$, the call option in portfolio A is exercised and portfolio A is worth F_T. If $F_T \leqslant X$, the call is not exercised and portfolio A is worth X. The value of portfolio A at time T is therefore

$$\max(F_T,\ X)$$

In portfolio B the cash can be invested at the risk-free rate to grow to F_0 at time T. The put option provides a payoff of $\max(X - F_T,\ 0)$. The futures contract provides a payoff of $F_T - F_0$.[1] The value of portfolio B at time T is therefore

$$F_0 + (F_T - F_0) + \max(X - F_T,\ 0) = \max(F_T,\ X)$$

Because the two portfolios have the same value at time T and there are no early exercise opportunities, it follows that they are worth the same today. The value of portfolio A today is

$$c + Xe^{-rT}$$

[1] This analysis assumes that a futures contract is like a forward contract and settled at the end of its life rather than on a day-to-day basis.

where c is the price of the call futures option. The marking-to-market process ensures that the futures contract in portfolio B is worth zero today. Portfolio B is therefore worth

$$p + F_0 e^{-rT}$$

where p is the price of the put futures option. Hence

$$c + Xe^{-rT} = p + F_0 e^{-rT} \tag{13.1}$$

Example

> Suppose that the price of a European call option on silver futures for delivery in six months is \$0.56 per ounce when the exercise price is \$8.50. Assume that the silver futures price for delivery in six months is currently \$8.00, and the risk-free interest rate for an investment that matures in six months is 10% per annum. From a rearrangement of equation (13.1), the price of a European put option on silver futures with the same maturity and exercise date as the call option is
>
> $$0.56 + 8.50e^{-0.1\times 6/12} - 8.00e^{-0.1\times 6/12} = 1.04$$

For American futures, the put–call parity relationship is (see Problem 13.19)

$$F_0 e^{-rT} - X < C - P < F_0 - Xe^{-rT} \tag{13.2}$$

13.5 BOUNDS FOR FUTURES OPTIONS

The put–call parity relationship in equation (13.1) provides bounds for European call and put options. Because the price of a put, p, cannot be negative, it follows from equation (13.1) that

$$c + Xe^{-rT} \geqslant F_0 e^{-rT}$$

or

$$c \geqslant (F_0 - X)e^{-rT} \tag{13.3}$$

Similarly, because the price of a call option cannot be negative, it follows from equation (13.1) that

$$Xe^{-rT} \leqslant F_0 e^{-rT} + p$$

or

$$p \geqslant (X - F_0)e^{-rT} \tag{13.4}$$

These bounds are similar to the ones derived for European stock options in Chapter 8. The prices of European call and put options are very close to their lower bounds when the options are deep in the money. To see why this is so, we return to the put–call parity relationship in equation (13.1). When a call option is deep in the money, the corresponding put option is deep out of the money. This means that p is very close to zero. The difference between c and its lower bound equals p, so that the price of the call option must be very close to its lower bound. A similar argument applies to put options.

Because American futures options can be exercised at any time, we must have

$$C \geqslant F_0 - X$$

and

$$P \geqslant X - F_0$$

Thus, if interest rates are positive, the lower bound for an American option price is always higher than the lower bound for a European option. This is because there is always some chance that an American futures option will be exercised early.

13.6 VALUATION OF FUTURES OPTIONS USING BINOMIAL TREES

This section uses a binomial tree approach similar to that developed in Chapter 10 to price futures options. A key difference between futures options and stock options is that there are no up-front costs when a futures contract is entered into.

Suppose that the current futures price is 30 and that it is considered that it will move either up to 33 or down to 28 over the next month. Consider a one-month call option on the futures with a strike price of 29. The situation is as indicated in Figure 13.1. If the futures price proves to be 33, the payoff from the option is 4 and the value of the futures contract is 3. If the futures price proves to be 28, the payoff from the option is zero and the value of the futures contract is −2.

To set up a riskless hedge, we consider a portfolio consisting of a short position in one options contract and a long position in Δ futures contracts. If the futures price moves up to 33, the value of the portfolio is $3\Delta - 4$; if it moves down to 28, the value of the portfolio is -2Δ. The portfolio is riskless when these are the same—that is, when

$$3\Delta - 4 = -2\Delta$$

or $\Delta = 0.8$.

For this value of Δ, we know the portfolio will be worth $3 \times 0.8 - 4 = -1.6$ in one month. Assume a risk-free interest rate of 6%. The value of the portfolio today must be

$$-1.6e^{-0.06\times 1/12} = -1.592$$

The portfolio consists of one short option and Δ futures contracts. Because the value of the futures contract today is zero, the value of the option today must be 1.592.

A Generalization

We can generalize this analysis by considering a futures price that starts at F_0 and is anticipated to rise to F_0u or move down to F_0d over the time period T. We consider an option maturing at time T and suppose that its payoff is f_u if the futures price moves up and f_d if it moves down. The situation is summarized in Figure 13.2.

The riskless portfolio in this case consists of a short position in one option combined

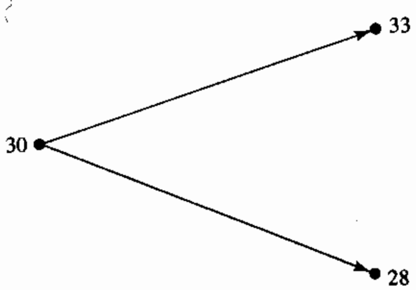

Figure 13.1 Futures price movements in numerical example

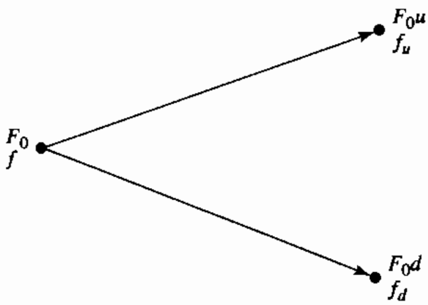

Figure 13.2 Futures price and option price in general situation

with a long position in Δ futures contracts where

$$\Delta = \frac{f_u - f_d}{F_0u - F_0d}$$

The value of the portfolio at time T is then always

$$(F_0u - F_0)\Delta - f_u$$

Denoting the risk-free interest rate by r, we obtain the value of the portfolio today:

$$[(F_0u - F_0)\Delta - f_u]e^{-rT}$$

Another expression for the present value of the portfolio is $-f$, where f is the value of the option today. It follows that

$$-f = [(F_0u - F_0)\Delta - f_u]e^{-rT}$$

Substituting for Δ and simplifying reduces this equation to

$$f = e^{-rT}[pf_u + (1 - p)f_d] \tag{13.5}$$

where

$$p = \frac{1 - d}{u - d} \tag{13.6}$$

In the numerical example considered previously (see Figure 13.1), $u = 1.1$, $d = 0.9333$, $r = 0.06$, $T = 1/12$, $f_u = 4$, and $f_d = 0$. From equation (13.6),

$$p = \frac{1 - 0.9333}{1.1 - 0.9333} = 0.4$$

and from equation (13.5),

$$f = e^{-0.06 \times 1/12}[0.4 \times 4 + 0.6 \times 0] = 1.592$$

This result agrees with the answer obtained for this example earlier.

13.7 A FUTURES PRICE AS AN ASSET PROVIDING A YIELD

There is a general result that makes the analysis of futures options analogous to the analysis of options on a stock paying a dividend yield. This result is that futures prices behave in the same way as a stock paying a dividend yield at the domestic risk-free rate r.

One clue that this might be so is given by comparing equations (13.5) and (13.6) with equations (12.7) and (12.8). The two sets of equations are identical when we set $q = r$. Another clue is that the lower bounds for futures options prices and the put–call parity relationship for futures options prices are the same as those for options on a stock paying a dividend yield at rate q when the stock price is replaced by the futures price and $q = r$.

We can understand the result by noting that a futures contract requires zero investment. In a risk-neutral world the expected profit from holding a position in an investment that costs zero to set up must be zero. The expected payoff from a futures contract in a risk-neutral world must therefore be zero. It follows that the expected growth rate of the futures price in a risk-neutral world must be zero. As pointed out in Chapter 12, a stock paying a dividend at rate q grows at an expected rate of $r - q$ in a risk-neutral world. If we set $q = r$, the expected growth rate of the stock price is zero, making it analogous to a futures price.

13.8 BLACK'S MODEL FOR VALUING FUTURES OPTIONS

European futures options can be valued using equations (12.4) and (12.5) with $q = r$. Fischer Black was the first to show this in a paper published in 1976. The underlying assumption is that futures prices have the same lognormal property that we assumed for stock prices in Chapter 11. The European call price, c, and the European put price, p, for a futures option are given by equations (12.4) and (12.5) with S_0 replaced by F_0 and $q = r$:

$$c = e^{-rT}[F_0 N(d_1) - XN(d_2)] \tag{13.7}$$

$$p = e^{-rT}[XN(-d_2) - F_0 N(-d_1)] \tag{13.8}$$

where

$$d_1 = \frac{\ln(F_0/X) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(F_0/X) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

and σ is the volatility of the futures price. When the cost of carry and the convenience yield are functions only of time, it can be shown that the volatility of the futures price is the same as the volatility of the underlying asset. Note that Black's formula does not require the options contract and the futures contract to mature at the same time.

Example

Consider a European put futures option on crude oil. The time to maturity is four months, the current futures price is \$20, the exercise price is \$20, the risk-free interest rate is 9% per annum, and the volatility of the futures price is 25% per annum. In this case $F_0 = 20$, $X = 20$, $r = 0.09$, $T = 4/12$, $\sigma = 0.25$, and $\ln(F_0/X) = 0$, so that

$$d_1 = \frac{\sigma\sqrt{T}}{2} = 0.07216$$

$$d_2 = -\frac{\sigma\sqrt{T}}{2} = -0.07216$$

$$N(-d_1) = 0.4712, \qquad N(-d_2) = 0.5288$$

and the put price p is given by

$$p = e^{-0.09\times 4/12}(20 \times 0.5288 - 20 \times 0.4712) = 1.12$$

or \$1.12.

13.9 COMPARISON OF FUTURES OPTION AND SPOT OPTION PRICES

The payoff from a European spot call option with strike price X is

$$\max(S_T - X,\ 0)$$

where S_T is the spot price at the option's maturity. The payoff from a European futures call option with the same strike price is

$$\max(F_T - X,\ 0)$$

where F_T is the futures price at the option's maturity. If the European futures option matures at the same time as the futures contract, $F_T = S_T$ and the two options are in theory equivalent. If the European call futures option matures before the futures contract, it is worth more than the corresponding spot option in a normal market (where futures prices are higher than spot prices) and less than the corresponding spot option in an inverted market (where futures prices are lower than spot prices).[2]

Similarly, a European futures put option is worth the same as its spot option counterpart when the futures option matures at the same time as the futures contract. If the European put futures option matures before the futures contract, it is worth less than the corresponding spot option in a normal market and more than the corresponding spot option in an inverted market.

Results for American Options

Traded futures options are in practice usually American. Assuming that the risk-free rate of interest, r, is positive, there is always some chance that it will be optimal to exercise an American futures option early. American futures options are therefore worth more than their European counterparts. We will look at numerical procedures for valuing American futures options in Chapter 17.

It is not generally true that an American futures option is worth the same as the corresponding American option on the underlying asset when the futures and options contracts have the same maturity. Suppose, for example, that there is a normal market with futures prices consistently higher than spot prices prior to maturity. This is the case with most stock indices, gold, silver, low-interest currencies, and some commodities. An

[2] The spot option "corresponding" to a futures option is defined here as one with the same strike price and the same expiration date.

American call futures option must be worth more than the corresponding American call option on the underlying asset. The reason is that in some situations the futures option will be exercised early, in which case it will provide a greater profit to the holder. Similarly, an American put futures option must be worth less than the corresponding American put option on the underlying asset. If there is an inverted market with futures prices consistently lower than spot prices, as is the case with high-interest currencies and some commodities, the reverse must be true. American call futures options are worth less than the corresponding American call option on the underlying asset, whereas American put futures options are worth more than the corresponding American put option on the underlying asset.

The differences just described between American futures options and American asset options hold true when the futures contract expires later than the options contract as well as when the two expire at the same time. In fact, the differences tend to be greater, the later the futures contract expires.

13.10 SUMMARY

Futures options require delivery of the underlying futures contract on exercise. When a call is exercised, the holder acquires a long futures position plus a cash amount equal to the excess of the futures price over the strike price. Similarly, when a put is exercised the holder acquires a short position plus a cash amount equal to the excess of the strike price over the futures price. The futures contract that is delivered usually expires slightly later than the option.

A futures price behaves in the same way as a stock that provides a dividend yield equal to the risk-free rate, r. This means that the results produced in Chapter 12 for options on stock paying a dividend yield apply to futures options if we replace the stock price by the futures price and set the dividend yield equal to the risk-free interest rate.

Pricing formulas for European futures options were first produced by Fischer Black in 1976. They assume that the futures price has a constant volatility, so that the futures price is lognormally distributed at the expiration of the option.

If we assume that the two expiration dates are the same, a European futures option is worth exactly the same as the corresponding European option on the underlying asset. This is not true of American options. If the futures market is normal, an American call futures is worth more than the American call on the underlying asset, whereas an American put futures is worth less than the American put on the underlying asset. If the futures market is inverted, the reverse is true.

Suggestions for Further Reading

Black, F. "The Pricing of Commodity Contracts." *Journal of Financial Economics* 3 (1976): 167–79.

Brenner, M., G. Courtadon, and M. Subrahmanyam. "Options on Spot and Options on Futures." *Journal of Finance* 40 (December 1985): 1303–17.

Ramaswamy, K., and S. M. Sundaresan. "The Valuation of Options on Futures Contracts." *Journal of Finance* 40 (December 1985): 1319–40.

Wolf, A. "Fundamentals of Commodity Options on Futures." *Journal of Futures Markets* 2 (1982): 391–408.

Quiz (Answers at End of Book)

13.1. Explain the difference between a call option on yen and a call option on yen futures.

13.2. Why are options on bond futures more actively traded than options on bonds?

13.3. "A futures price is like a stock paying a dividend yield." What is the dividend yield?

13.4. A futures price is currently 50. At the end of six months it will be either 56 or 46. The risk-free interest rate is 6% per annum. What is the value of a six-month European call option with a strike price of 50?

13.5. How does the put–call parity formula for a futures option differ from put–call parity for an option on a non-dividend-paying stock?

13.6. Consider an American futures call option where the futures contract and the option contract expire at the same time. Under what circumstances is the futures option worth more than the corresponding American option on the underlying asset?

13.7. Calculate the value of a five-month European put futures option when the futures price is \$19, the strike price is \$20, the risk-free interest rate is 12% per annum, and the volatility of the futures price is 20% per annum.

Questions and Problems (Answers in Solutions Manual)

13.8. Suppose you buy a put option contract on October gold futures with a strike price of \$400 per ounce. Each contract is for the delivery of 100 ounces. What happens if you exercise when the October futures price is \$380?

13.9. Suppose you sell a call option contract on April live cattle futures with a strike price of 70 cents per pound. Each contract is for the delivery of 40,000 pounds. What happens if the contract is exercised when the futures price is 75 cents?

13.10. Consider a two-month call futures option with a strike price of 40 when the risk-free interest rate is 10% per annum. The current futures price is 47. What is a lower bound for the value of the futures option if it is (a) European and (b) American?

13.11. Consider a four-month put futures option with a strike price of 50 when the risk-free interest rate is 10% per annum. The current futures price is 47. What is a lower bound for the value of the futures option if it is (a) European and (b) American?

13.12. A futures price is currently 60. It is known that over each of the next two three-month periods it will either rise by 10% or fall by 10%. The risk-free interest rate is 8% per annum. What is the value of a six-month European call option on the futures with a strike price of 60? If the call were American, would it ever be worth exercising it early?

13.13. In Problem 13.12 what is the value of a six-month European put option on futures with a strike price of 60? If the put were American, would it ever be worth exercising it early? Verify that the call prices calculated in Problem 13.12 and the put prices calculated here satisfy put–call parity relationships.

13.14. A futures price is currently 25, its volatility is 30% per annum, and the risk-free interest rate is 10% per annum. What is the value of a nine-month European call on the futures with a strike price of 26?

13.15. A futures price is currently 70, its volatility is 20% per annum, and the risk-free interest rate is 6% per annum. What is the value of a five-month European put on the futures with a strike price of 65?

13.16. Suppose that a one-year futures price is currently 35. A one-year European call option and a one-year European put option on the futures with a strike price of 34 are both priced at 2 in the market. The risk-free interest rate is 10% per annum. Identify an arbitrage opportunity.

13.17. "The price of an at-the-money European call futures option always equals the price of a similar at-the-money European put futures option." Explain why this statement is true.

13.18. Suppose that a futures price is currently 30. The risk-free interest rate is 5% per annum. A three-month American call futures option with a strike price of 28 is worth 4. Calculate bounds for the price of a three-month American put futures option with a strike price of 28.

13.19. Show that if C is the price of an American call option on a futures contract when the strike price is X and the maturity is T, and P is the price of an American put on the same futures contract with the same strike price and exercise date,

$$F_0e^{-r(T-t)} - X < C - P < F_0 - Xe^{-r(T-t)}$$

where F_0 is the futures price and r is the risk-free rate. Assume that $r > 0$ and that there is no difference between forward and futures contracts. (*Hint*: Use an analogous approach to that indicated for Problem 12.12.)

Assignment Questions

13.20. A futures price is currently 40. It is known that at the end of three months the price will be either 35 or 45. What is the value of a three-month European call option on the futures with a strike price of 42 if the risk-free interest rate is 7% per annum?

13.21. Calculate the implied volatility of soybean futures prices from the following information concerning a European put on soybean futures:

Current futures price	525
Exercise price	525
Risk-free rate	6% per annum
Time to maturity	5 months
Put price	20

13.22. Use the DerivaGem software to calculate implied volatilities for the May options on corn futures in Table 13.3. Assume the futures prices in Table 2.2 apply and that the risk-free rate is 5% per annum. Treat the options as American and use 100 time steps. The options mature on April 21, 2001. Can you draw any conclusions from the pattern of implied volatilities you obtain?