CHAPTER 14 Volatility Smiles

How close are the market prices of options to those predicted by Black–Scholes? Do traders really use Black–Scholes when determining a price for an option? Are the probability distributions of asset prices really lognormal? What research has been carried out to test the validity of the Black–Scholes formulas? In this chapter we answer these questions. We explain that traders do use the Black–Scholes model—but not in exactly the way that Black and Scholes originally intended. This is because they allow the volatility used to price an option to depend on its strike price and time to maturity.

A plot of the implied volatility of an option as a function of its strike price is known as a *volatility smile*. In this chapter we describe the volatility smiles that traders use in equity and foreign currency markets. We explain the relationship between a volatility smile and the probability distribution being assumed for the future asset price. We also discuss how option traders allow volatility to be a function of option maturity and how they use volatility matrices as pricing tools. The final part of the chapter summarizes some of the work researchers have carried out to test Black–Scholes.

14.1 PUT–CALL PARITY REVISITED

Put–call parity, which we explained in Chapters 8 and 12, provides a good starting point for understanding volatility smiles. It is an important relationship between the price, c, of a European call and the price, p, of a European put:

$$p + S_0 e^{-qT} = c + Xe^{-rT} \tag{14.1}$$

The call and the put have the same strike price, X, and time to maturity, T. The variable, S_0, is the price of the underlying asset today, r is the risk-free interest rate for maturity T, and q is the yield on the asset.

A key feature of the put–call parity relationship is that it is based on a relatively simple arbitrage argument. It does not require any assumption about the future probability distribution of the asset price. It is true both when the asset price distribution is lognormal and when it is not lognormal.

Suppose that, for a particular value of the volatility, p_{bs} and c_{bs} are the values of European put and call options calculated using the Black–Scholes model. Suppose further that p_{mkt} and c_{mkt} are the market values of these options. Because put–call parity holds for the Black–Scholes model, we must have

$$p_{bs} + S_0e^{-qT} = c_{bs} + Xe^{-rT}$$

Because it also holds for the market prices

$$p_{mkt} + S_0e^{-qT} = c_{mkt} + Xe^{-rT}$$

Subtracting these two equations

$$p_{bs} - p_{mkt} = c_{bs} - c_{mkt} \tag{14.2}$$

This shows that the dollar pricing error when the Black–Scholes model is used to price a European put option should be exactly the same as the dollar pricing error when it is used to price a European call option with the same strike price and time to maturity.

Suppose that the implied volatility of the put option is 22%. This means that $p_{bs} = p_{mkt}$ when a volatility of 22% is used in the Black–Scholes model. From equation (14.2), it follows that $c_{bs} = c_{mkt}$ when this volatility is used. The implied volatility of the call is, therefore, also 22%. This argument shows that the implied volatility of a European call option is always the same as the implied volatility of a European put option when the two have the same strike price and maturity date. To put this another way, for a given strike price and maturity, the correct volatility to use in conjunction with the Black–Scholes model to price a European call should always be the same as that used to price a European put. This is also approximately true for American options. It follows that when traders refer to the relationship between implied volatility and strike price, or to the relationship between implied volatility and maturity, they do not need to state whether they are talking about calls or puts. The relationship is the same for both.

Example

The value of the Australian dollar is $0.60. The risk-free interest rate is 5% per annum in the United States and 10% per annum in Australia. The market price of a European call option on the Australian dollar with a maturity of one year and a strike price of $0.59 is 0.0236. DerivaGem shows that the implied volatility of the call is 14.5%. For there to be no arbitrage, the put–call parity relationship in equation (14.1) must apply with q equal to the foreign risk-free rate. The price, p, of a European put option with a strike price of $0.59 and maturity of one year therefore satisfies:

$$p + 0.60e^{-0.10\times 1} = 0.0236 + 0.59e^{-0.05\times 1}$$

so that $p = 0.0419$. DerivaGem shows that, when the put has this price, its implied volatility is also 14.5%. This is what we expect from the analysis just given.

14.2 FOREIGN CURRENCY OPTIONS

The volatility smile used by traders to price foreign currency options has the general form shown in Figure 14.1. The volatility is relatively low for at-the-money options. It becomes progressively higher as an option moves either in the money or out of the money.

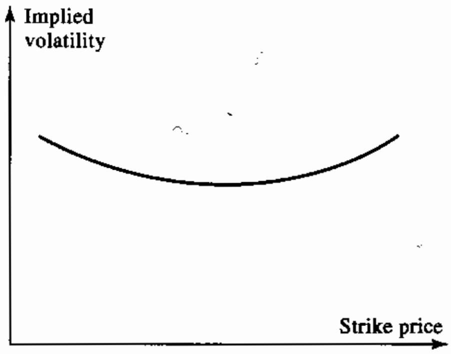

Figure 14.1 Volatility smile for foreign currency options

The volatility smile in Figure 14.1 corresponds to the probability distribution shown by the solid line in Figure 14.2. We will refer to this as the *implied distribution*. A lognormal distribution with the same mean and standard deviation as the implied distribution is shown by the dashed line in Figure 14.2. It can be seen that the implied distribution has fatter tails than the lognormal distribution.[1]

To see that Figure 14.1 and 14.2 are consistent with each other, consider first a deep-out-of-the-money call option with a high strike price of X_2. This option pays off only if the exchange rate proves to be above X_2. Figure 14.2 shows that the probability of this is higher for the implied probability distribution than for the lognormal distribution. We, therefore, expect the implied distribution to give a relatively high price for the option. A relatively high price leads to a relatively high implied volatility—and this is exactly what

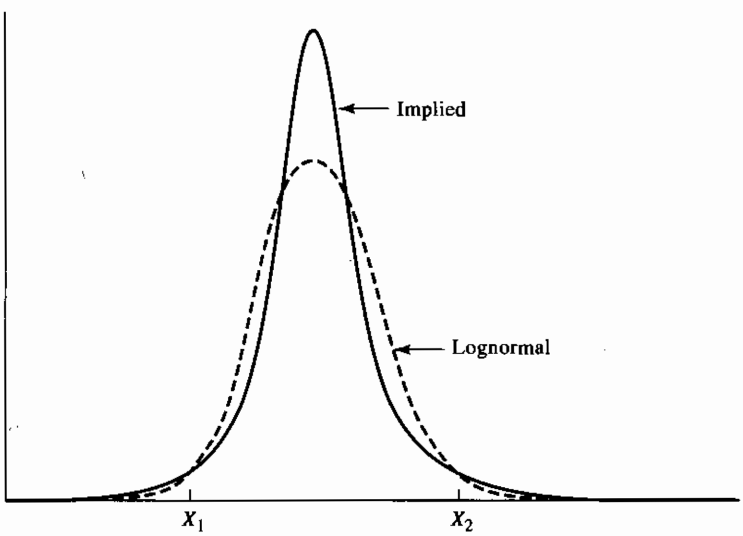

Figure 14.2 Implied distribution and lognormal distribution for foreign currency options

[1] This is known as kurtosis. Note that in addition to having a fatter tail, the implied distribution is more "peaked." Both small and large movements in the exchange rate are more likely than with the lognormal distribution. Intermediate movements are less likely.

we observe in Figure 14.1 for the option. The two figures are therefore consistent with each other for high strike prices. Consider next a deep-out-of-the-money put option with a low strike price of X_1. This option pays off only if the exchange rate proves to be below X_1. Figure 14.2 shows that the probability of this is also higher for implied probability distribution than for the lognormal distribution. We therefore expect the implied distribution to give a relatively high price, and a relatively high implied volatility, for this option as well. Again, this is exactly what we observe in Figure 14.1.

Reason for the Smile in Foreign Currency Options

We have just shown that the smile used by traders for foreign currency options implies that they consider that the lognormal distribution understates the probability of extreme movements in exchange rates. To test whether they are right, we examined the daily movements in 12 different exchange rates over a 10-year period. As a first step we calculated the standard deviation of daily percentage change in each exchange rate. We then noted how often the actual percentage change exceeded one standard deviation, two standard deviations, and so on. Finally, we calculated how often this would have happened if the the percentage changes had been normally distributed. (The lognormal model implies that percentage changes are almost exactly normally distributed over a one-day time period.) The results are shown in Table 14.1.[2]

Daily changes exceed three standard deviations on 1.34% of days. The lognormal model predicts that this should happen on only 0.27% of days. Daily changes exceed four, five, and six standard deviations on 0.29%, 0.08%, and 0.03% of days, respectively. The lognormal model predicts that we should hardly ever observe this happening. The table, therefore, provides evidence to support the existence of fat tails and volatility smile used by traders.

Why are exchange rates not lognormally distributed? Two of the conditions for an asset price to have a lognormal distribution are

1. The volatility of the asset is constant.
2. The price of the asset changes smoothly with no jumps.

In practice, neither of these conditions is satisfied for an exchange rate. The volatility of an exchange rate is far from constant, and exchange rates frequently exhibit jumps.[3] It

Table 14.1 Percent of days when daily exchange rate moves are greater than one, two,..., six standard deviations (S.D. = standard deviation of daily change)

	Real world	Lognormal model
> 1 S.D.	25.04	31.73
> 2 S.D.	5.27	4.55
> 3 S.D.	1.34	0.27
> 4 S.D.	0.29	0.01
> 5 S.D.	0.08	0.00
> 6 S.D.	0.03	0.00

[2] This table is taken from J. C. Hull and A. White, "Value at Risk When Daily Changes in Market Variables Are Not Normally Distributed." *Journal of Derivatives*, 5(3) (spring 1998): 9–19.

[3] Often the jumps are in response to the actions of central banks.

turns out that the effect of both a nonconstant volatility and jumps is that extreme outcomes become more likely.

The impact of jumps and nonconstant volatility depends on the option maturity. The percentage impact of a nonconstant volatility on prices becomes more pronounced as the maturity of the option is increased, but the volatility smile created by the nonconstant volatility usually becomes less pronounced. The percentage impact of jumps on both prices and the volatility smile becomes less pronounced as the maturity of the option is increased. When we look at sufficiently long-dated options, jumps tend to get "averaged out" so that the stock price distribution when there are jumps is almost indistinguishable from the one obtained when the stock price changes smoothly.

14.3 EQUITY OPTIONS

The volatility smile used by traders to price equity options (both those on individual stocks and those on stock indices) has the general form shown in Figure 14.3. This is sometimes referred to as a *volatility skew*. The volatility decreases as the strike price increases. The volatility used to price a low strike price option (that is, a deep-out-of-the-money put or a deep-in-the-money call) is significantly higher than that used to price a high-strike-price option (that is, a deep-in-the-money put or a deep-out-of-the-money call).

The volatility smile for equity options corresponds to the implied probability distribution given by the solid line in Figure 14.4. A lognormal distribution with the same mean and standard deviation as the implied distribution is shown by the dotted line. It can be seen that the implied distribution has a fatter left tail and thinner right tail than the lognormal distribution.

To see that Figures 14.3 and 14.4 are consistent with each other, we proceed as for Figures 14.1 and 14.2 and consider options that are deep-out-of-the-money. From

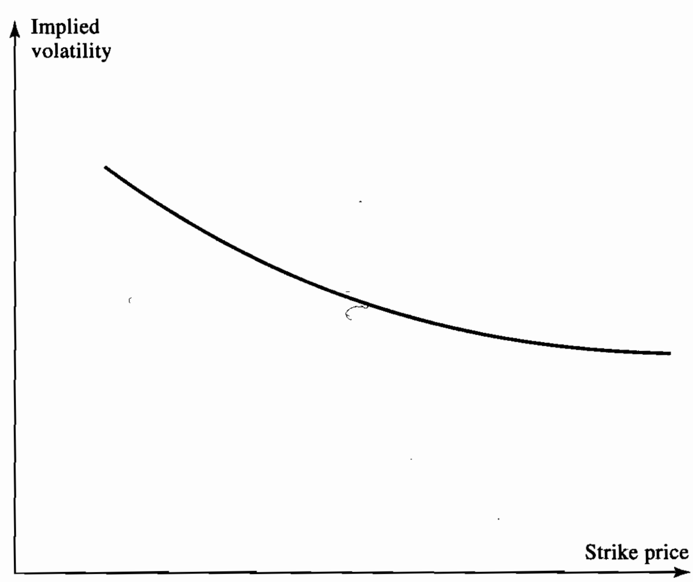

Figure 14.3 Volatility smile for equities

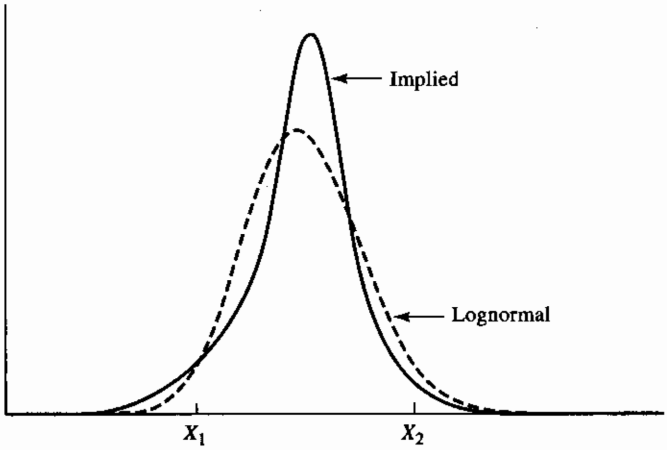

Figure 14.4 Implied distribution and lognormal distribution for equity options

Figure 14.4 a deep-out-of-the-money call with a strike price of X_2 has a lower price when the implied distribution is used than when the lognormal distribution is used. This is because the option pays off only if the stock price proves to be above X_2, and the probability of this is lower for the implied probability distribution than for the lognormal distribution. Therefore, we expect the implied distribution to give a relatively low price for the option. A relatively low price leads to a relatively low implied volatility—and this is exactly what we observe in Figure 14.3 for the option. Consider next a deep-out-of-the-money put option with a strike price of X_1. This option pays off only if the stock price proves to be below X_1. Figure 14.3 shows that the probability of this is higher for implied probability distribution than for the lognormal distribution. We therefore expect the implied distribution to give a relatively high price, and a relatively high implied volatility, for this option. Again, this is exactly what we observe in Figure 14.3.

The Reason for the Smile in Equity Options

One possible explanation for the smile in equity options concerns leverage. As a company's equity declines in value, the company's leverage increases. This means that the equity becomes more risky and its volatility increases. As a company's equity increases in value, leverage decreases. The equity then becomes less risky and its volatility decreases. This argument shows that we can expect the volatility of equity to be a decreasing function of price and is consistent with Figures 14.3 and 14.4.

It is interesting that the pattern in Figure 14.3 for equities has existed only since the stock market crash of October 1987. Prior to October 1987 implied volatilities were much less dependent on strike price. This has led Mark Rubinstein to suggest that one reason for the pattern in Figure 14.3 may be "crashophobia." Traders are concerned about the possibility of another crash similar to October 1987, and they price options accordingly. This explanation has some validity. Although the evidence is somewhat mixed, it appears that the implied probability distribution for a stock price has fatter left tails than the probability distribution calculated from empirical data on stock market returns. Also the skew became more pronounced after the October 1997 and August 1998 declines.

14.4 THE VOLATILITY TERM STRUCTURE AND VOLATILITY MATRICES

In addition to a volatility smile, traders use a volatility term structure when pricing options. This means that the volatility used to price an at-the-money option depends on the maturity of the option. Volatility tends to be an increasing function of maturity when short-dated volatilities are historically low. This is because there is then an expectation that volatilities will increase. Similarly, volatility tends to be an decreasing function of maturity when short-dated volatilities are historically high. This is because there is then an expectation that volatilities will decrease.

Volatility matrices combine volatility smiles with the volatility term structure to tabulate the volatilities appropriate for pricing an option with any strike price and any maturity. An example of a volatility matrix that might be used for foreign currency options is shown in Table 14.2.

One dimension of a volatility matrix is strike price; the other is time to maturity. The main body of the matrix shows implied volatilities calculated from the Black–Scholes model. At any given time, some of the entries in the matrix are likely to correspond to options for which reliable market data are available. The implied volatilities for these options are calculated directly from their market prices and entered into the table. The rest of the matrix is determined using linear interpolation.

When a new option has to be valued, financial engineers look up the appropriate volatility in the table. For example, when valuing a nine-month option with a strike price of 1.05, a financial engineer would interpolate between 13.4 and 14.0 to obtain a volatility of 13.7%. This is the volatility that would be used in the Black–Scholes formula (or in the binomial tree model, which we will discuss further in Chapter 17).

The shape of the volatility smile depends on the option maturity. As illustrated in Table 14.2, the smile tends to become less pronounced as the option maturity increases. Define T as the time to maturity and F_0 as the forward price of the asset. Some financial engineers choose to define the volatility smile as the relationship between implied volatility and

$$\frac{1}{\sqrt{T}}\ln\frac{X}{F_0}$$

rather than as the relationship between the implied volatility and X. The smile is then usually much less dependent on the time to maturity.[4]

Table 14.2 Volatility matrix

	Strike price				
	0.90	0.95	1.00	1.05	1.10
1 month	14.2	13.0	12.0	13.1	14.5
3 month	14.0	13.0	12.0	13.1	14.2
6 month	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0

[4] For a discussion of this approach see S. Natenberg *Option Pricing and Volatility: Advanced Trading Strategies and Techniques*, 2nd edn. McGraw-Hill, 1994; R. Tompkins *Options Analysis: A State of the Art Guide to Options Pricing*, Burr Ridge, IL: Irwin, 1994.

The Role of the Model

How important is the pricing model if traders are prepared to use a different volatility for every deal? It can be argued that the Black–Scholes model is no more than a sophisticated interpolation tool used by traders for ensuring that an option is priced consistently with the market prices of other actively traded options. If traders stopped using Black–Scholes and switched to another plausible model, the matrix of volatilities would change and the shape of the smile would change. But arguably, the dollar prices quoted in the market would not change appreciably.

14.5 WHEN A SINGLE LARGE JUMP IS ANTICIPATED

Suppose that a stock price is currently \$50 and an important news announcement in a few days is expected to either increase the stock price by \$8 or reduce it by \$8. (This announcement could concern the outcome of a takeover attempt or the verdict in an important lawsuit.) The probability distribution of the stock price in, say, three months might then consist of a mixture of two lognormal distributions, the first corresponding to favorable news, the second to unfavorable news. The situation is illustrated in Figure 14.5. The solid line shows the mixtures-of-lognormals distribution for the stock price in three months; the dashed line shows a lognormal distribution with the same mean and standard deviation as this distribution. Assume that favorable news and unfavorable news are equally likely.[5] Assume also that after the news (favorable or unfavorable) the volatility will be constant at 20% for three month.

Consider a three-month European call option on the stock with a strike price of \$50. We assume that the risk-free interest rate is 5% per annum. Because the news announcement is expected very soon, the value of the option assuming favorable news can be calculated from the Black–Scholes formula with $S_0 = 58$, $X = 50$, $r = 5\%$, $\sigma = 20\%$, and $T = 0.25$. It is 8.743. Similarly, the value of the option assuming unfavorable news can be calculated from the Black–Scholes formula with $S_0 = 42$, $X = 50$, $r = 5\%$, $\sigma = 20\%$, and $T = 0.25$. It is 0.101. The value of the call option today

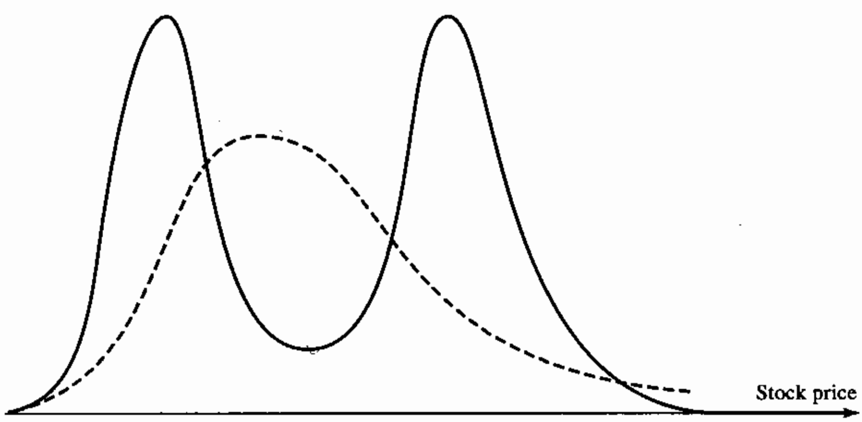

Figure 14.5 Effect of a single large jump. The solid line is the true distribution; the dashed line is the lognormal distribution

[5] Strictly speaking, we are assuming that the probabilities are equally likely in a risk-neutral world.

Table 14.3 Implied volatilities in situation where an important announcement is imminent

Strike price ($)	Call price if good news ($)	Call price if bad news ($)	Call price today ($)	Implied volatility (%)
35	23.435	7.471	15.453	30.95
40	18.497	3.169	10.833	35.46
45	13.565	0.771	7.168	39.94
50	8.743	0.101	4.422	41.48
55	4.546	0.008	2.277	39.27
60	1.764	0.000	0.882	35.66
65	0.494	0.000	0.247	32.50

should therefore be

$$0.5 \times 8.743 + 0.5 \times 0.101 = 4.422$$

The implied volatility calculated from this option price is 41.48%.

A similar calculation can be made for other strike prices and a volatility smile constructed. The results of doing this are shown in Table 14.3, and the volatility smile is shown in Figure 14.6.[6] It turns out that we are in the opposite situation to that of Figure 14.1. At-the-money options have higher volatilities than either out-of-the-money or in-the-money options.

14.6 EMPIRICAL RESEARCH

A number of problems arise in carrying out empirical research to test the Black–Scholes and other option pricing models.[7] The first problem is that any statistical hypothesis about how options are priced has to be a joint hypothesis to the effect that (1) the

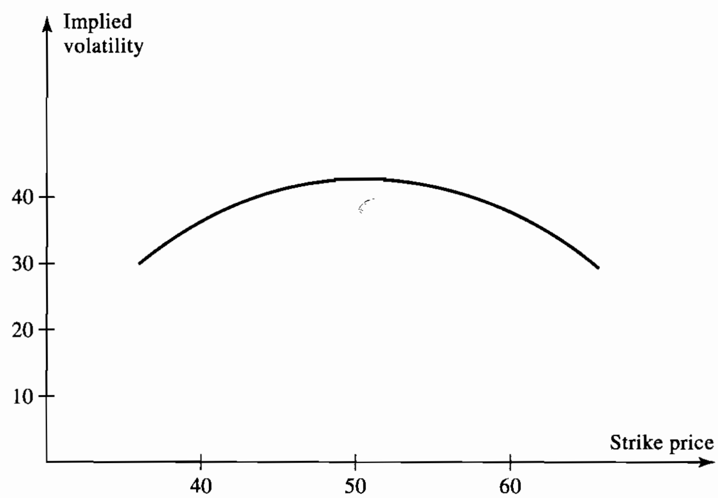

Figure 14.6 Volatility smile for situation in Table 14.3

[6] In this case, the smile is a frown!

[7] See the end-of-chapter references for citations to the studies reviewed in this section.

option pricing formula is correct and (2) markets are efficient. If the hypothesis is rejected, it may be the case that (1) is untrue, (2) is untrue, or both (1) and (2) are untrue. A second problem is that the stock price volatility is an unobservable variable. One approach is to estimate the volatility from historical stock price data. Alternatively, implied volatilities can be used in some way. A third problem for the researcher is to make sure that data on the stock price and option price are synchronous. For example, if the option is thinly traded, it is not likely to be acceptable to compare closing option prices with closing stock prices. The closing option price might correspond to a trade at 1:00 p.m., whereas the closing stock price corresponds to a trade at 4:00 p.m.

Black and Scholes and Galai have tested whether it is possible to make excess returns above the risk-free rate of interest by buying options that are undervalued by the market (relative to the theoretical price) and selling options that are overvalued by the market (relative to the theoretical price). Black and Scholes used data from the over-the-counter options market where options are dividend protected. Galai used data from the Chicago Board Options Exchange (CBOE) where options are not protected against the effects of cash dividends. Galai used Black's approximation as described in Section 11.10 to incorporate the effect of anticipated dividends into the option price. Both studies showed that, in the absence of transactions costs, significant excess returns over the risk-free rate can be obtained by buying undervalued options and selling overvalued options. But, it is possible that these excess returns are available only to market makers and that, when transactions costs are considered, they vanish.

A number of researchers have chosen to make no assumptions about the behavior of stock prices and have tested whether arbitrage strategies can be used to make a riskless profit in options markets. Garman provides a computational procedure for finding any arbitrage possibilities that exist in a given situation. One frequently cited study by Klemkosky and Resnick tests whether the relationship in equation (8.8) is ever violated. It concludes that some small arbitrage profits are possible from using the relationship. These are due mainly to the overpricing of American calls.

Chiras and Manaster carried out a study using CBOE data to compare a weighted implied volatility from options on a stock at a point in time with the volatility calculated from historical data. They found that the former provide a much better forecast of the volatility of the stock price during the life of the option. We can conclude that option traders are using more than just historical data when determining future volatilities. Chiras and Manaster also tested to see whether it was possible to make above-average returns by buying options with low implied volatilities and selling options with high implied volatilities. The strategy showed a profit of 10% per month. The Chiras and Manaster study can be interpreted as providing good support for the Black–Scholes model and showing that the CBOE was inefficient in some respects.

MacBeth and Merville tested the Black–Scholes model using a different approach. They looked at different call options on the same stock at the same time and compared the volatilities implied by the option prices. The stocks chosen were AT&T, Avon, Kodak, Exxon, IBM, and Xerox, and the time period considered was the year 1976. They found that implied volatilities tended to be relatively high for in-the-money options and relatively low for out-of-the-money options. A relatively high implied volatility is indicative of a relatively high option price, and a relatively low implied volatility is indicative of a relatively low option price. Therefore, if it is assumed that Black–Scholes prices at-the-money options correctly, it can be concluded that out-of-the-money (high strike price) call options are overpriced by Black–Scholes and in-the-money (low strike price) call options are underpriced by Black–Scholes. These effects become more pronounced as the time to maturity increases and the degree to which the option is in or out of the money increases. MacBeth and Merville's results are consistent with Figure 14.3. The results were confirmed by Lauterbach and Schultz in a later study concerned with the pricing of warrants.

Rubinstein has done a great deal of research similar to that of MacBeth and Merville. No clear-cut pattern emerged from his early research, but the research in his 1994 paper and joint 1996 paper with Jackwerth gives results consistent with Figure 14.3. Options with low strike prices have much higher volatilities than those with high strike prices. As mentioned previously in the chapter, leverage and the resultant negative correlation between volatility and stock price may partially account for the finding. It is also possible that investors fear a repeat of the crash of 1987.

A number of authors have researched the pricing of options on assets other than stocks. For example, Shastri and Tandon and Bodurtha and Courtadon have examined the market prices of currency options; in another paper, Shastri and Tandon have examined the market prices of futures options; and Chance has examined the market prices of index options.

In most cases, the mispricing by Black–Scholes is not sufficient to present profitable opportunities to investors when transactions costs and bid–offer spreads are taken into account. When profitable opportunities are searched for, it is important to bear in mind that, even for a market maker, some time must elapse between a profitable opportunity being identified and action being taken. This delay, even if it is only to the next trade, can be sufficient to eliminate the profitable opportunity.

14.7 SUMMARY

The Black–Scholes model and its extensions assume that the probability distribution of the underlying asset at any given future time is lognormal. This assumption is not the one made by traders. They assume the probability distribution of an equity price has a fatter left tail and thinner right tail than the lognormal distribution. They also assume that the probability of an exchange rate has a fatter right tail and a fatter left tail than the lognormal distribution.

Traders use volatility smiles to allow for nonlognormality. The volatility smile defines the relationship between the implied volatility of an option and its strike price. For equity options, the volatility smile tends to be downward sloping. This means that out-of-the-money puts and in-the-money calls tend to have high implied volatilities whereas out-of-the-money calls and in-the-money puts tend to have low implied volatilities. For foreign currency options, the volatility smile is U-shaped. Deep-out-of-the-money and deep-in-the-money options have higher implied volatilities than at-the-money options.

Often traders also use a volatility term structure. The implied volatility of an option then depends on its life. When volatility smiles and volatility term structures are combined, they produce a volatility matrix. This defines implied volatility as a function of both the strike price and the time to maturity.

Suggestions for Further Reading

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Quiz (Answers at End of Book)

14.1. What pattern of implied volatilities is likely to be observed when

a. Both tails of the stock price distribution are thinner than those of the lognormal distribution?

b. The right tail is fatter, and the left tail is thinner, than that of a lognormal distribution?

14.2. What pattern of implied volatilities is likely to be observed for six-month options when the volatility is uncertain and positively correlated to the stock price?

14.3. What pattern of implied volatilities are likely to be caused by jumps in the underlying asset price? Is the pattern likely to be more pronounced for a six-month option than for a three-month option?

14.4. A call and put option have the same strike price and time to maturity. Show that the difference between their prices should be the same for any option pricing model.

14.5. Explain carefully why Figure 14.4 is consistent with Figure 14.3.

14.6. The market price of a European call is \$3.00 and its Black–Scholes price is \$3.50. The Black Scholes price of a European put option with the same strike price and time to maturity is \$1.00. What should the market price of this option be? Explain the reasons for your answer.

14.7. A stock price is currently \$20. Tomorrow, news is expected to be announced that will either increase the price by \$5 or decrease the price by \$5. What are the problems in using Black–Scholes to value one-month options on the stock?

Questions and Problems (Answers in Solutions Manual)

14.8. What are the major problems in testing a stock option pricing model empirically?

14.9. Suppose that a central bank's policy is to allow an exchange rate to fluctuate between 0.97 and 1.03. What pattern of implied volatilities for options on the exchange rate would you expect to see?

14.10. Option traders sometimes refer to deep-out-of-the-money options as being options on volatility. Why do you think they do this?

14.11. A European call option on a certain stock has a strike price of \$30, a time to maturity of one year, and an implied volatility of 30%. A European put option on the same stock has a strike price of \$30, a time to maturity of one year, and an implied volatility of 33%. What is the arbitrage opportunity open to a trader? Does the arbitrage work only when the lognormal assumption underlying Black–Scholes holds? Explain the reasons for your answer carefully.

14.12. Suppose that the result of a major lawsuit affecting Microsoft is due to be announced tomorrow. Microsoft's stock price is currently \$60. If the ruling is favorable to Microsoft, the stock price is expected to jump to \$75. If it is unfavorable, the stock is expected to jump to \$50. What is the risk-neutral probability of a favorable ruling? Assume that the volatility of Microsoft's stock will be 25% for six months after the ruling if the ruling is favorable and 40% if it is unfavorable. Use DerivaGem to calculate the relationship between implied volatility and strike price for six-month European options on Microsoft today. Microsoft does not pay dividends. Assume that the six-month risk-free rate is 6%. Consider call options with strike prices of 30, 40, 50, 60, 70, and 80.

4.13. An exchange rate is currently 0.8000. The volatility of the exchange rate is quoted as 12% and interest rates in the two countries are the same. Using the lognormal assumption, estimate the probability that the exchange rate in three months will be

(a) less than 0.7000, (b) between 0.7000 and 0.7500, (c) between 0.7500 and 0.8000, (d) between 0.8000 and 0.8500, (e) between 0.8500 and 0.9000, and (f) greater than 0.9000. Based on the volatility smile usually observed in the market for exchange rates, which of these estimates would you expect to be too low and which would you expect to be too high?

14.14. A stock price is $40. A six-month European call option on the stock with a strike price of $30 has an implied volatility of 35%. A six-month European call option on the stock with a strike price of $50 has an implied volatility of 28%. The six-month risk-free rate is 5% and no dividends are expected. Explain why the two implied volatilities are different. Use DerivaGem to calculate the prices of the two options. Use put–call parity to calculate the prices of six-month European put options with strike prices of $30 and $50. Use DerivaGem to calculate the implied volatilities of these two put options.

14.15. "The Black–Scholes model is used by traders as an interpolation tool." Discuss this view.

Assignment Questions

14.16. A company's stock is selling for $4. The company has no outstanding debt. Analysts consider the liquidation value of the company to be at least $300,000 and there are 100,000 shares outstanding. What volatility smile would you expect to see?

14.17. A company is currently awaiting the outcome of a major lawsuit. This is expected to be known within one month. The stock price is currently $20. If the outcome is positive, the stock price is expected to be $24 at the end of one month. If the outcome is negative, it is expected to be $18 at this time. The one-month risk-free interest rate is 8% per annum.
  a. What is the risk-neutral probability of a positive outcome?
  b. What are the values of one-month call options with strike prices of $19, $20, $21, $22, and $23?
  c. Use DerivaGem to calculate a volatility smile for one-month call options.
  d. Verify that the same volatility smile is obtained for one-month put options.

14.18. A futures price is currently $40. The risk-free interest rate is 5%. Some news is expected tomorrow that will cause the volatility over the next three months to be either 10% or 30%. There is a 60% chance of the first outcome and a 40% chance of the second outcome. Use DerivaGem to calculate a volatility smile for three-month options.

14.19. Data for a number of foreign currencies are provided on the author's Web site:

http://www.rotman.utoronto.ca/~hull

Choose a currency and use the data to produce a table similar to Table 14.1.

14.20. Data for a number of stock indices are provided on the author's Web site:

http://www.rotman.utoronto.ca/~hull

Choose an index and test whether a three standard deviation down movement happens more often than a three standard deviation up movement.