CHAPTER 19

Exotic Options and Other Nonstandard Products

The derivatives we have covered in the first 18 chapters of this book are what are termed *plain vanilla products.* They have standard well-defined properties and trade actively. Their prices or implied volatilities are quoted by exchanges or by brokers on a regular basis. One of the exciting aspects of the over-the-counter derivatives market is the number of nonstandard (or exotic) products that have been created by financial engineers. Although they are usually a relatively small part of its portfolio, these exotic products are important to an investment bank because they are generally much more profitable than plain vanilla products.

Exotic products come about for a number of reasons. Sometimes they meet a genuine hedging need in the market; sometimes there are tax, accounting, legal, or regulatory reasons why corporate treasurers find exotic products attractive; sometimes the products are designed to reflect a corporate treasurer's view on potential future movements in particular market variables; occasionally an exotic product is designed by an investment bank to appear more attractive than it is to an unwary corporate treasurer.

We start by considering exotic options. These are variations on the standard call and put options that we have focused on in Chapters 7 to 17. We then move on to look at mortgage-backed securities, which have become an important feature of the United States interest rate derivatives market. Finally we describe some nonstandard swap products. It should be stressed that this chapter does not provide a comprehensive list of all exotic products that exist. Its objective is to give a flavor for the range of instruments that have been developed.

19.1 EXOTIC OPTIONS

In this section we describe a number of different types of exotic options that large investment banks offer on underlying assets such as stocks, stock indices, and currencies. We use a categorization similar to that in an excellent series of articles written by Eric Reiner and Mark Rubinstein for *RISK* magazine in 1991 and 1992.

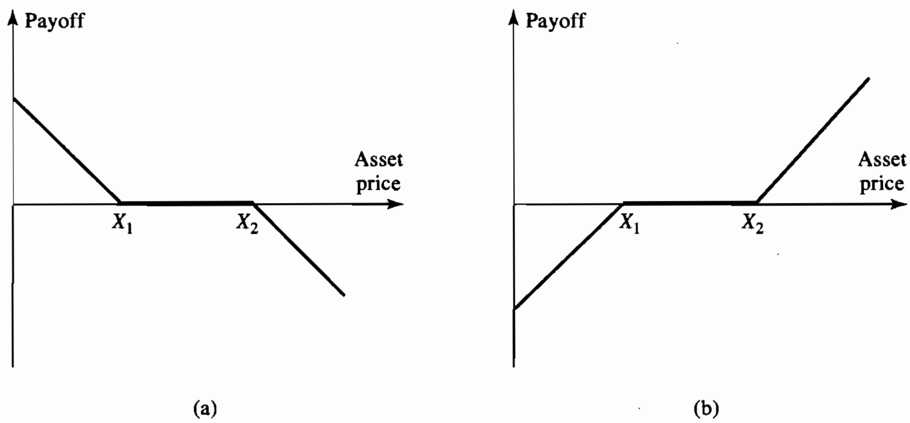

Figure 19.1 Payoffs from (a) short and (b) long range-forward contract

Asian, barrier, binary, chooser, compound, and lookback options can all be valued using DerivaGem.[1]

Packages

A *package* is a portfolio consisting of standard European calls, standard European puts, forward contracts, cash, and the underlying asset itself. We discussed a number of different types of packages in Chapter 9: bull spreads, bear spreads, butterfly spreads, calendar spreads, straddles, strangles, and so on.

Often a package is structured by traders so that it has zero cost initially. An example is a *range-forward contract*.[2] A short range-forward contract consists of a long position in a put with a low strike price, X_1, and a short position in a call with a high strike price, X_2. It guarantees that the underlying asset can be sold for a price between X_1 and X_2 at the maturity of the options. A long range-forward contract consists of a short position in a put with the low strike price, X_1, and a long position in a call with the high strike price, X_2. It guarantees that the underlying asset can be purchased for a price between X_1 and X_2 at the maturity of the options. The price of the call equals the price of the put when the contract is initiated. Figure 19.1 shows the payoff from short and long range-forward contracts. As X_1 and X_2 are moved closer to each other, the price that will be received or paid for the asset at maturity becomes more certain. In the limit when $X_1 = X_2$, the range-forward contract becomes a regular forward contract.

Nonstandard American Options

In a standard American option, exercise can take place at any time during the life of the option, and the exercise price is always the same. In practice, the American options that are traded in the over-the-counter market do not always have these standard features.

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[1] The procedures used by the market to value all the options described in this section are covered in J. C. Hull, *Options, Futures, and Other Derivatives*, 4th edn. (Upper Saddle River, N.J.: Prentice Hall, 2000), Chapter 18.

[2] Other names used for a range-forward contract are zero-cost collar, flexible forward, cylinder option, option fence, min-max, and forward band.

For example:

1. Early exercise may be restricted to certain dates. The instrument is then known as a *Bermudan option.*
2. Early exercise may be allowed during only part of the life of the option.
3. The strike price may change during the life of the option.

The warrants issued by corporations on their own stock often have some of these features. For example, in a seven-year warrant, exercise might be possible on particular dates during years three to seven, with the strike price being $30 during years three and four, $32 during the next two years, and $33 during the final year.

Nonstandard American options can usually be valued using a binomial tree. At each node, the test (if any) for early exercise is adjusted to reflect the terms of the option.

Forward Start Options

Forward start options are options that will start at some time in the future. They are sometimes used in employee incentive schemes. For example a company might promise to give an employee a certain number of options on its stock at certain times in the future. Usually the agreement states that the options will be at the money when they are issued.

When the underlying asset provides no income, an at-the-money forward start option is (using the assumptions underlying Black–Scholes) worth the same as a regular at-the-money option with the same life. For example, an at-the-money option that will start in three years and mature in five years is worth the same as a two-year at-the-money option initiated today. (See Problem 19.10.)

Compound Options

Compound options are options on options. There are four main types of compound options: a call on a call, a put on a call, a call on a put, and a put on a put. Compound options have two strike prices and two exercise dates. Consider, for example, a call on a call. On the first exercise date, T_1, the holder of the compound option is entitled to pay the first strike price, X_1, and receive a call option. The call option gives the holder the right to buy the underlying asset for the second strike price, X_2, on the second exercise date, T_2. The compound option will be exercised on the first exercise date only if the value of the second option on that date is greater than the first strike price. A compound option is generally much more sensitive to volatility than a plain vanilla option.

Chooser Options

A *chooser option* (sometimes referred to as an *as you like it option*) has the feature that, after a specified period of time, the holder can choose whether the option is a call or a put. Suppose that the time when the choice is made is T_1. The value of the chooser option at this time is

$$\max(c,\ p)$$

where c is the value of the call underlying the option and p is the value of the put underlying the option.

If the options underlying the chooser option are both European and have the same strike price, put–call parity can be used to provide a valuation formula. Suppose that S_1

is the stock price at time T_1, X is the strike price, T_2 is the maturity of the options, and r is the risk-free interest rate. Put–call parity implies that

$$\begin{aligned}\max(c,\ p) &= \max(c,\ c + Xe^{-r(T_2-T_1)} - S_1 e^{-q(T_2-T_1)}) \\ &= c + e^{-q(T_2-T_1)} \max(0,\ Xe^{-(r-q)(T_2-T_1)} - S_1)\end{aligned}$$

This shows that the chooser option is a package consisting of

1. A call option with strike price X and maturity T_2
2. $e^{-q(T_2-T_1)}$ put options with strike price $Xe^{-(r-q)(T_2-T_1)}$ and maturity T_1

As such, it can readily be valued.

Barrier Options

Barrier options are options where the payoff depends on whether the underlying asset's price reaches a certain level during a certain period of time. A number of different types of barrier options regularly trade in the over-the-counter market. They are attractive to some market participants because they are less expensive than the corresponding regular options. Barrier options can be classified as either *knock-out options* or *knock-in options*. A knock-out option ceases to exist when the underlying asset price reaches a certain level; a knock-in option comes into existence only when the underlying asset price reaches a certain level.

There are four types of knock-out options. An *up-and-out call* option is a regular European call option that ceases to exist as soon as the asset price reaches a barrier level. The barrier level is greater than the asset price at the time the option is initiated. A *down-and-out call* is defined similarly except that the barrier level is below the asset price at the time the option is initiated. An *up-and-out put* and *down-and-out put* are defined similarly.

There are similarly four types of knock-in options. An *up-and-in call* option is a regular European call option that starts to exist as soon as the asset price reaches a barrier level. The barrier level is greater than the asset price when the option is initiated. A *down-and-in call* is similar except that the barrier level is below the asset price when the option is initiated. An *up-and-in put* and a *down-and-in put* are defined analogously.

There are relationships between the prices of barrier options and regular options. For example, the price of a down-and-out call option plus the price of a down-and-in call option must equal the price of a regular European option. Similarly, the price of a down-and-out put option plus the price of a down-and-in put option must equal the price of a regular European option.

Barrier options often have quite different properties from regular options. For example, sometimes vega is negative. Consider an up-and-out call option when the asset price is close to the barrier level. As volatility increases the probability that the barrier will be hit increases. As a result, a volatility increase causes a price decrease.

In determining whether a barrier is hit, sometimes the price is observed on a more or less continuous basis.[3] On other occasions the terms on the contract state that the price is observed periodically; for example, once a day at 12 noon.

[3] One way to track whether a barrier is reached from below (above) is to send a limit order to an exchange to sell (buy) the asset at a certain price and see whether the order is filled.

Binary Options

Binary options are options with discontinuous payoffs. A simple example of a binary option is a *cash-or-nothing call*. This pays off nothing if the stock price ends up below the strike price at time T and pays a fixed amount, Q, if it ends up above the strike price. In a risk-neutral world, the probability of the stock price being above the strike price at the maturity of an option is, with our usual notation, $N(d_2)$. The value of a cash-or-nothing call is therefore $Qe^{-rT}N(d_2)$. A *cash-or-nothing put* is defined analogously to a cash-or-nothing call. It pays off Q if the stock price is below the strike price and nothing if it is above the strike price. The value of a cash-or-nothing put is $Qe^{-rT}N(-d_2)$.

Another type of binary option is an *asset-or-nothing call*. This pays off nothing if the underlying stock price ends up below the strike price and pays an amount equal to the stock price itself if it ends up above the strike price. With our usual notation, the value of an asset-or-nothing call is $S_0e^{-qT}N(d_1)$. An *asset-or-nothing put* pays off nothing if the underlying stock price ends up above the strike price and an amount equal to the stock price if it ends up below the strike price. The value of an asset-or-nothing put is $S_0e^{-qT}N(-d_1)$.

A regular European call option is equivalent to a long position in an asset-or-nothing call and a short position in a cash-or-nothing call where the cash payoff on the cash-or-nothing call equals the strike price. Similarly, a regular European put option is equivalent to a long position in a cash-or-nothing put and a short position in an asset-or-nothing put where the cash payoff on the cash-or-nothing put equals the strike price.

Lookback Options

The payoffs from lookback options depend on the maximum or minimum stock price reached during the life of the option. The payoff from a European-style lookback call is the amount that the final stock price exceeds the minimum stock price achieved during the life of the option. The payoff from a European-style lookback put is the amount by which the maximum stock price achieved during the life of the option exceeds the final stock price.

A lookback call is a way that the holder can buy the underlying asset at the lowest price achieved during the life of the option. Similarly, a lookback put is a way that the holder can sell the underlying asset at the highest price achieved during the life of the option. The underlying asset in a lookback option is often a commodity. The frequency with which the asset price is observed for the purposes of computing the maximum or minimum is important and must be specified in the contract.

Shout Options

A *shout option* is a European option where the holder can "shout" to the writer at one time during its life. At the end of the life of the option, the option holder receives either the usual payoff from a European option or the intrinsic value at the time of the shout, whichever is greater. Suppose the strike price is \$50 and the holder of a call shouts when the price of the underlying asset is \$60. If the final asset price is less than \$60, the holder receives a payoff of \$10. If it is greater than \$60, the holder receives the excess of the asset price over \$50.

A shout option has some of the same features as a lookback option, but is considerably less expensive. It can be valued by noting that, if the option is shouted at a time τ when the asset price is S_τ, the payoff from the option is

$$\max(0,\ S_T - S_\tau) + (S_\tau - X)$$

where, as usual, X is the strike price and S_T is the asset price at time T. The value at time τ if the option is shouted is, therefore, the present value of $S_\tau - X$ plus the value of a European option with strike price S_τ. This allows a binomial tree to be used to value the option.

Asian Options

Asian options are options where the payoff depends on the average price of the underlying asset during at least some part of the life of the option. The payoff from an *average price call* is $\max(0,\ S_{ave} - X)$ and that from an *average price put* is $\max(0,\ X - S_{ave})$, where S_{ave} is the average value of the underlying asset calculated over a predetermined averaging period. Average price options are less expensive than regular options and are arguably more appropriate than regular options for meeting some of the needs of corporate treasurers. Suppose that a U.S. corporate treasurer expects to receive a cash flow of 100 million Australian dollars spread evenly over the next year from the company's Australian subsidiary. The treasurer is likely to be interested in an option that guarantees that the average exchange rate realized during the year is above some level. An average price put option can achieve this more effectively than regular put options.

Another type of Asian option is an average strike option. An *average strike call* pays off $\max(0,\ S_T - S_{ave})$ and an *average strike put* pays off $\max(0,\ S_{ave} - S_T)$. Average strike options can guarantee that the average price paid for an asset in frequent trading over a period of time is not greater than the final price. Alternatively, it can guarantee that the average price received for an asset in frequent trading over a period of time is not less than the final price.

Options to Exchange One Asset for Another

Options to exchange one asset for another (sometimes referred to as *exchange options*) arise in various contexts. An option to buy yen with Australian dollars is, from the point of view of a U.S. investor, an option to exchange one foreign currency asset for another foreign currency asset. A stock tender offer is an option to exchange shares in one stock for shares in another stock.

An option to obtain the better or worse of two assets is closely related to an exchange option. It is a position in one of the assets combined with an option to exchange it for the other asset:

$$\min(U_T,\ V_T) = V_T - \max(V_T - U_T,\ 0)$$

$$\max(U_T,\ V_T) = U_T + \max(V_T - U_T,\ 0)$$

Options Involving Several Assets

Options involving two or more risky assets are sometimes referred to as *rainbow options*. One example is the bond futures contract traded on the CBOT described in Chapter 5. The party with the short position is allowed to choose between a large number of different bonds when making delivery. Another example is a LIBOR–Contingent FX option. This is a foreign currency option whose payoff occurs only if a prespecified interest rate is within a certain range at maturity.

Probably the most common example of an option involving several assets is a *basket option*. This is an option where the payoff is dependent on the value of a portfolio (or basket) of assets. The assets are usually either individual stocks or stock indices or currencies.

19.2 MORTGAGE-BACKED SECURITIES

One feature of the United States interest rate derivatives market is the active trading in *mortgage-backed securities*. A mortgage-backed security (MBS) is created when a financial institution decides to sell part of its residential mortgage portfolio to investors. The mortgages are put into a pool and investors acquire a stake in the pool by buying units. The units are known as mortgage-backed securities. A secondary market is usually created for the units so that investors can sell them to other investors as desired. An investor who owns units representing X percent of a certain pool is entitled to X percent of the principal and interest cash flows received from the mortgages in the pool.

The mortgages in a pool are generally guaranteed by a government-related agency such as the Government National Mortgage Association (GNMA) or the Federal National Mortgage Association (FNMA) so that investors are protected against defaults. This makes an MBS sound like a regular fixed-income security issued by the government. However, there is a critical difference between an MBS and a regular fixed-income investment. The mortgages in an MBS pool have prepayment privileges and these can be quite valuable to the householder. In the United States mortgages typically last for 25 years and can be prepaid at any time. This means that the householder has a 25-year American-style option to put the mortgage back to the lender at its face value.

In practice, prepayments on mortgages occur for a variety of reasons. Sometimes interest rates have fallen and the owner of the house decides to refinance at a lower rate of interest. On other occasions a mortgage is prepaid simply because the house is being sold. A critical element in valuing an MBS is the determination of the *prepayment function*. This function describes expected prepayments on the underlying pool of mortgages at a particular time in terms of interest rates and other relevant variables.

A prepayment function is very unreliable as a predictor of actual prepayment experience for an individual mortgage. When many similar mortgage loans are combined in the same pool, there is a "law of large numbers" effect at work, and prepayments can be predicted from an analysis of historical data more accurately. As already mentioned, prepayments are not always motivated by pure interest rate considerations. Nevertheless, prepayments tend to be more likely when interest rates are low than when they are high. This means that investors should require a higher rate of interest on an MBS than on other fixed-income securities because there is a tendency for the cash received from prepayments to be reinvested at low rates.

Collateralized Mortgage Obligations

The MBSs described so far are sometimes referred to as *pass-throughs*. All investors receive the same return and bear the same prepayment risk. Not all mortgage-backed securities work in this way. In a *collateralized mortgage obligation* (CMO) the investors are divided into a number of classes, and rules are developed for determining how principal repayments are channeled to different classes.

As an example of a CMO consider an MBS with investors divided into three classes: class A, class B, and class C. All the principal repayments (both those that are scheduled and those that are prepayments) are channeled to class A investors until investors in this class have been completely paid off. Principal repayments are then channeled to class B investors until these investors have been completely paid off. Finally, principal repayments are channeled to class C investors. In this situation class A investors bear the most prepayment risk. The class A securities can be expected to last less long than the class B securities, which in turn can be expected to last less long than the class C securities.

The objective of this type of structure is to create classes of securities that are more attractive to institutional investors than those created by the simpler pass-through MBS. The prepayment risks assumed by the different classes depend on the par value in each class. For example, class C bears very little prepayment risk if the par values in classes A, B, and C are 400, 300, and 100, respectively. It bears rather more prepayment risk if the par values in the classes are 100, 200, and 500.

IOs and POs

In a *stripped MBS*, principal payments are separated from interest payments. All principal payments are channeled to one class of security, known as a *principal only* (PO). All interest payments are channeled to another class of security, known as an *interest only* (IO). Both IOs and POs are risky investments. As prepayment rates increase, a PO becomes more valuable and an IO becomes less valuable. As prepayment rates decrease, the reverse happens. In a PO a fixed amount of principal is returned to the investor, but the timing is uncertain. A high rate of prepayments on the underlying pool leads to the principal being received early (which is, of course, good news for the holder of the PO). A low rate of prepayments on the underlying pool delays the return of the principal and reduces the yield provided by the PO. In an IO the total of the cash flows received by the investor is not certain. The higher the rate of prepayments, the lower the total cash flows received by the investor, and vice versa.

19.3 NONSTANDARD SWAPS

We discussed plain vanilla interest rate swaps in Chapter 6. These are agreements to exchange interest at the LIBOR rate for interest at a fixed rate. Table 6.3 gives a confirmation for a hypothetical plain vanilla swap. In this section we describe a number of nonstandard swap agreements.[4]

Variations on the Vanilla Deal

Many interest rate swaps involve relatively minor changes being made to the plain vanilla structure in Table 6.3. In some swaps the notional principal changes with time. Swaps where the notional principal is an increasing function of time are known as *step-up swaps*. Swaps where the notional principal is a decreasing function of time are known as

[4] The valuation of many of the swaps described here is described in J. C. Hull, *Options, Futures, and Other Derivatives*, 4th edn. (Upper Saddle River, N.J.: Prentice Hall, 2000), Chapters 19–22.

Table 19.1 Extract from confirmation for a hypothetical swap where the principal and payment frequency are different on the two sides

Trade date	4-January-2001
Effective date	11-January-2001
Business day convention (all dates)	Following business day
Holiday calendar	U.S.
Termination date	11-January-2006
Fixed amounts	
Fixed rate payer	Microsoft
Fixed rate notional principal	USD 100 million
Fixed rate	6% per annum
Fixed rate day count convention	Actual/365
Fixed rate payment dates	Each 11-July and 11-January commencing 11-July, 2001, up to and including 11-January, 2006
Floating amounts	
Floating rate payer	Citibank
Floating rate notional principal	USD 120 million
Floating rate	USD 1 month LIBOR
Floating rate day count convention	Actual/360
Floating rate payment dates	11-July, 2001, and the 11th of each month thereafter up to and including 11-January, 2006

amortizing swaps. Step-up swaps could be useful for a construction company that intends to borrow increasing amounts of money at floating rates to finance a particular project and wants to swap it to fixed-rate funding. An amortizing swap could be used by a company that has fixed-rate borrowings with a certain prepayment schedule and wants to swap them to borrowings at a floating rate.

The principal can be different on the two sides of a swap. Also, the frequency of payment can be different. This is illustrated in Table 19.1, which shows a hypothetical swap between Microsoft and Citibank where the notional principal is $120 million on the floating side and $100 million on fixed side. Payments are made every month on the floating side and every six months on the fixed side.

The floating reference rate for a swap is not always LIBOR. For example, in some swaps it is the three-month Treasury bill rate. A *basis swap* consists of exchanging cash flows calculated using one floating reference rate for cash flows calculated using another floating reference rate. An example would be a swap where the three-month Treasury bill rate plus 30 basis points is exchanged for three-month LIBOR with both being applied to a principal of $100 million. A basis swap could be used for risk management by a financial institution whose assets and liabilities are dependent on different floating reference rates.

Compounding Swaps

Another variation on the plain vanilla swap is a *compounding swap*. Table 19.2 gives an example. There is only one payment date for both the floating-rate payments and the

Table 19.2 Extract from confirmation for a hypothetical compounding swap

Trade date	4-January-2001
Effective date	11-January-2001
Holiday calendar	U.S.
Business day convention (all dates)	Following business day
Termination date	11-January-2006
Fixed amounts	
Fixed rate payer	Microsoft
Fixed rate notional principal	USD 100 million
Fixed rate	6% per annum
Fixed rate day count convention	Actual/365
Fixed rate payment date	11-January, 2006
Fixed-rate compounding	Applicable at 6.3%
Fixed-rate compounding dates	Each 11-July and 11-January commencing 11-July, 2001, up to and including 11-July, 2005
Floating amounts	
Floating rate payer	Citibank
Floating rate notional principal	USD 100 million
Floating rate	USD 6 month LIBOR plus 20 basis points
Floating rate day count convention	Actual/360
Floating rate payment date	11-January, 2006
Floating rate Compounding	Applicable at LIBOR plus 10 basis points
Floating-rate compounding dates	Each 11-July and 11-January commencing 11-July, 2001, up to and including 11-July, 2005

fixed-rate payments. This is at the end of the life of the swap. The floating rate of interest is LIBOR plus 20 basis points. Instead of being paid, the interest is compounded forward until the end of the life of the swap at a rate of LIBOR plus 10 basis points. The fixed rate of interest is 6%. Instead of being paid this interest is compounded forward at a fixed rate of interest of 6.3% until the end of the swap.

Currency Swaps

We introduced currency swaps in Chapter 6. They enable an interest rate exposure in one currency to be swapped for an interest rate exposure in another currency. Usually two principals are specified, one in each currency. The principals are exchanged at both the beginning and the end of the life of the swap, as described in Section 6.4.

Suppose that the currencies involved in a currency swap are U.S. dollars (USD) and British pounds (GBP). In a fixed-for-fixed currency swap, a fixed rate of interest is specified in each currency. The payments on one side are determined by applying the fixed rate of interest in USD to the USD principal; the payments on the other side are determined by applying the fixed rate of interest in GBP to the GBP principal.

Another popular type of currency swap is floating-for-floating. In this the payments on one side are determined by applying the USD LIBOR (possibly with a spread added) to the USD principal; similarly the payments on the other side are determined by applying the GBP LIBOR (possibly with a spread added) to the GBP principal. A third type of swap is a cross-currency interest rate swap where a floating rate in one currency is exchanged for a fixed rate in another currency.

Valuation and Convexity Adjustments

In Chapter 6 we explained that plain vanilla interest rate and currency swaps can be valued by assuming that interest rates in the future will equal the corresponding forward interest rates observed in the market today. The nonstandard swaps we have discussed so far can also be valued in this way. However, the next three types of swap that we will discuss cannot. They are valued by assuming that interest rates in the future will equal the corresponding forward interest rates observed in the market today plus an adjustment. The adjustment is known as a *convexity adjustment.*[5]

LIBOR-in-Arrears Swaps

A plain vanilla interest rate swap is designed so that the floating rate of interest observed on one payment date is paid on the next payment date. An alternative instrument that is sometimes traded is a *LIBOR-in-arrears swap*. In this, the floating rate paid on a payment date equals the rate observed on the payment date itself.

CMS and CMT Swaps

A constant maturity swap (CMS) is an interest rate swap where the floating rate equals the swap rate for a swap with a certain life. For example, the floating payments on a CMS swap might be made every six months at a rate equal to the five-year swap rate. Usually there is a lag so that the payment on a particular payment date is equal to the swap rate observed on the previous payment date. Suppose that rates are set at times $t_0, t_1, t_2, \ldots$ and payments are made at times $t_1, t_2, t_3, \ldots$. The floating payment at time t_{i+1} is

$$\delta_i L s_i$$

where $\delta_i = t_{i+1} - t_i$ and s_i is the five-year swap rate at time t_i.

Differential Swaps

A *differential swap*, sometimes referred to as a *diff swap*, is an interest rate swap where a floating interest rate is observed in one currency and applied to a principal in another currency. For example, a swap might involve the payments going one way being calculated as USD LIBOR applied to a USD principal and the payments going the other way being calculated as GBP LIBOR (plus or minus a spread) being applied to the same USD principal. Diff swaps are sometimes also referred to as *quantos*.

A diff swap is a "pure interest rate play." This distinguishes it from a regular floating-for-floating currency swap. The company paying GBP in our diff swap example gains if GBP LIBOR decreases relative to USD LIBOR and loses if the reverse happens. The value of a currency swap where GBP floating is exchanged for USD floating depends on exchange-rate movements as well as on interest rate movements in the two countries.

[5] For a discussion of these types of convexity adjustments, see J. C. Hull, *Options, Futures, and Other Derivatives*, 4th edn. (Upper Saddle River, N.J.: Prentice Hall, 2000), Chapter 20.

Table 19.3 Extract from confirmation for an equity swap

Trade date	4-January-2001
Effective date	11-January-2001
Business day convention (all dates)	Following business day
Holiday calendar	U.S.
Termination date	11-January-2006
Equity amounts	
Equity payer	Microsoft
Equity index	Total Return S&P 500 index
Equity payment	$100(I_1 - I_0)/I_0$ where I_1 is the index level on the payment date and I_0 is the index level on the immediately preceding payment date. In the case of the first payment date, I_0 is the index level on 11 January, 2001
Equity payment dates	Each 11-July and 11-January commencing 11-July, 2001, up to and including 11-January, 2006
Floating amounts	
Floating rate payer	Citibank
Floating rate notional principal	USD 100 million
Floating rate	USD 6 month LIBOR
Floating rate day count convention	Actual/360
Floating rate payment dates	Each 11-July and 11-January commencing 11-July, 2001, up to and including 11-January, 2006

Equity Swaps

In an equity swap one party promises to pay the return on an equity index on a notional principal and the other promises to pay a fixed or floating return on a notional principal. Equity swaps enable fund managers to increase or reduce their exposure to an index without buying and selling stock. An equity swap is a convenient way of packaging a series of forward contracts on an index to meet the needs of the market.

The equity index is usually a total return index where dividends are reinvested in the stocks comprising the index. An example of an equity swap is in Table 19.3. In this Microsoft pays the six-month return on the S&P 500 to Citibank and Citibank pays six-month LIBOR to Microsoft. The principal on either side of the swap is $100 million and payments are made every six months.

Accrual Swaps

Accrual swaps are swaps where the interest on one side accrues only when the floating reference rate is within a certain range. Sometimes the range remains fixed during the entire life of the swap; sometimes it is reset periodically.

As a simple example of an accrual swap, consider a deal where a fixed rate of 6% is exchanged for three-month LIBOR every quarter. The principal is $10 million and the fixed rate accrues only on days when three-month LIBOR is below 8% per annum.

Define n_1 as the number of business days in a quarter that the three-month LIBOR is below 8% and n_2 is the number of business days in the year. The payment made at the end of the quarter is

$$10{,}000{,}000 \times 0.06 \times \frac{n_1}{n_2}$$

For example when $n_1 = 25$ and $n_2 = 252$ the payment is \$59,524. In a regular swap the payment would be about $0.25 \times 0.06 \times 10{,}000{,}000$ or \$150,000.

Compared to a regular swap, fixed-rate payer saves $10{,}000{,}000 \times 0.06/252 = \$2{,}381$ for each day interest rates are above 8%. The fixed-rate payer's position can, therefore, be considered equivalent to a regular swap plus a series of binary options, one for each day of the life of the swap.

Cancelable Swaps

A cancelable swap is a plain vanilla interest rate swap where one side has the option to terminate on one or more payment dates. Terminating a swap is the same as entering into the offsetting (opposite) swap. Consider a swap between Microsoft and Citibank. If Microsoft has the option to cancel, it can regard the swap as a regular swap plus a long position in an option to enter into the offsetting swap. If Citibank has the cancelation option, Microsoft has a regular swap plus a short position in an option to enter into the same swap.

If there is only one termination date, a cancelable swap is the same as a regular swap plus a position in a European swap option. Consider, for example, a ten-year swap where Microsoft will receive 6% and pay LIBOR. Suppose that Microsoft has the option to terminate at the end of six years. The swap is a regular ten-year swap to receive 6% and pay LIBOR plus long position in a six-year European option to enter into a four-year swap where 6% is paid and LIBOR is received. (The latter is referred to as a 6×4 European option.) The standard market model for valuing European swap options is described in Chapter 18.

When the swap can be terminated on a number of different payment dates, it is a regular swap option plus a Bermudan-style swap option. Consider, for example, the situation where Microsoft has entered into a five-year swap with semiannual payments where 6% is received and LIBOR is paid. Suppose that the counterparty has the option to terminate on the swap on payment dates between year two and year five. The swap is a regular swap plus a short position in a Bermudan-style swap option where the Bermudan-style swap option is an option to enter into a swap that matures in five years and involves a fixed payment at 6% being received and a floating payment at LIBOR being paid. The swap can be exercised on any payment date between year two and year five.

Sometimes compounding swaps are cancelable. Typically the confirmation agreement states that on termination the floating-rate payer pays the compounded value of the floating amounts up to the termination date and the fixed rate pays the compounded value of the fixed payments up to the termination date.

Index Amortizing Swaps

A swap that was very popular in the United States in the mid-1990s is an *index amortizing swap* (sometimes also called an *indexed principal swap*). In this the principal reduces in a way dependent on the level of interest rates. The lower the interest rate, the greater the reduction in the principal. The fixed side of an indexed amortizing swap was originally designed to mirror, at least approximately, the return obtained by an investor from a mortgage-backed security. To an investor, the swap then has the effect of exchanging the return on a mortgage-backed security for a floating-rate return.

Commodity Swaps

Commodity swaps are now becoming increasingly popular. A company that consumes 100,000 barrels of oil per year could agree to pay \$2 million each year for the next 10 years and to receive in return $100{,}000S$, where S is the current market price of oil per barrel. The agreement would in effect lock in the company's oil cost at \$20 per barrel. An oil producer might agree to the opposite exchange, thereby locking in the price it realized for its oil at \$20 per barrel.

Volatility Swaps

A recent innovation in swap markets is a *volatility swap*. In this the payments depend on the volatility of a stock (or other asset). Suppose that the principal is L. On each payment date, one side pays $L\sigma$ where σ is the historical volatility calculated in the usual way by taking daily observations on the stock during the immediately preceding accrual period and the other side pays $L\sigma_K$ where σ_K is a constant prespecified volatility level. Variance swaps, correlation swaps, and covariance swaps are defined similarly.

Bizarre Deals

Some swaps involve the exchange of payments being calculated in quite bizarre ways. An example is provided by the so-called "5/30" swap entered into between Bankers Trust (BT) and Procter and Gamble (P&G) on November 2, 1993.[6] This was a five-year swap with semiannual payments. The notional principal was \$200 million. BT paid P&G 5.30% per annum. P&G paid BT the average 30-day CP (commercial paper) rate minus 75 basis points plus a spread. The average commercial paper rate was calculated by taking observations the 30-day commercial paper rate each day during the preceding accrual period and averaging them.

The spread is zero for the first payment date (May 2, 1994). For the remaining nine payment dates it is

$$\max\left[0, \frac{98.5\left(\dfrac{5\text{ yr CMT\%}}{5.78\%}\right) - (30\text{ yr TSY Price})}{100}\right]$$

In this, five-year CMT is the constant maturity Treasury yield (that is, the yield on a five-year Treasury note, as reported by the Federal Reserve). The 30-year TSY price is the midpoint of the bid and offer cash bond prices for the 6.25% Treasury bond maturing on August 2023. Note that the spread calculated from the formula is a decimal interest rate. It is not measured in basis points. If the formula gives 0.1, and the CP rate is 6% the rate paid by P&G is 15.25%.

P&G were hoping that the spread would be zero and the deal would enable them to

[6] See D. J. Smith, "Aggressive Corporate Finance: A Close Look at the Procter and Gamble–Bankers Trust Leveraged Swap." *Journal of Derivatives* 4(4) (summer 1997): 67–79.

exchange fixed-rate funding at 5.30% for funding at 75 basis points less than the commercial paper rate. In fact, interest rates rose sharply in early 1994, bond prices fell, and the swap proved very very expensive. This example is discussed further in Chapter 21. (See also Problem 19.17.)

19.4 SUMMARY

Exotic options are options with rules governing the payoffs that are not as straightforward than those for standard options. They provide corporate treasurers with a wide range of alternatives for achieving their objectives. Some exotic options are nothing more than portfolios of regular European and American calls and puts. Others are much more complicated.

Mortgage-backed securities are created when a financial institution decides to sell part of its residential portfolio of mortgages to investors. The mortgages are put in a pool and investors acquire a stake in the pool by buying units. The mortgages are guaranteed against defaults by a government agency, but investors are subject to prepayment risk. Often the return from a pool of mortgages is split into a number of components with different properties in an attempt to meet the needs of different types of investors.

Swaps have proved to be very versatile financial instruments, and many variations on the plain vanilla fixed-for-floating deal now exist. Some such as step-up swaps, amortizing swaps, compounding swaps, LIBOR in arrears swaps, diff swaps, and CMS/CMT swaps involve changes to the way payments are calculated or their timing. Others such as accrual swaps and cancelable swaps have embedded options.

Suggestions for Further Reading

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Quiz (Answers at End of Book)

19.1. Explain the difference between a forward start option and a chooser option.

19.2. Describe the payoff from a portfolio consisting of a lookback call and a lookback put with the same maturity.

19.3. Consider a chooser option where the holder has the right to choose between a European call and a European put at any time during a two-year period. The maturity dates and strike prices for the calls and puts are the same regardless of when the choice is made. Is it ever optimal to make the choice before the end of the two-year period? Explain your answer.

19.4. Suppose that c_1 and p_1 are the prices of a European average price call and a European average price put with strike price X and maturity T, c_2 and p_2 are the prices of a European average strike call and European average strike put with maturity T, and c_3 and p_3 are the prices of a regular European call and a regular European put with strike price X and maturity T. Show that

$$c_1 + c_2 - c_3 = p_1 + p_2 - p_3$$

19.5. Explain why IOs and POs have opposite sensitivities to the rate of prepayments.

19.6. Explain the relationship between a cancelable swap and a swap option.

19.7. The Canadian dollar LIBOR rate is 2% higher than the U.S. LIBOR rate for all maturities. A trader thinks that the spread between three-month U.S. LIBOR and three-month Canadian LIBOR will widen, but is unsure about how the exchange rate between the U.S. dollar and Canadian dollar will move. Explain how the trader could use a diff swap. Why would the trader prefer a diff swap to a floating-for-floating currency swap?

Questions and Problems (Answers in the Solutions Manual)

19.8. The text derives a decomposition of a particular type of chooser option into a call maturing at time t_2 and a put maturing at time t_1. By using put–call parity to obtain an expression for c instead of p, derive an alternative decomposition into a call maturing at time t_1 and a put maturing at time t_2.

19.9. Explain why a down-and-out put is worth zero when the barrier is greater than the strike price.

19.10. Prove that an at-the-money forward start option on a non-dividend-paying stock that will start in three years and mature in five years is worth the same as a two-year at-the-money option starting today.

19.11. Suppose that the strike price of an American call option on a non-dividend-paying stock grows at rate g. Show that if g is less than the risk-free rate, r, it is never optimal to exercise the call early.

19.12. Answer the following questions about compound options:

a. What put–call parity relationship exists between the price of a European call on a call and a European put on a call?

b. What put–call parity relationship exists between the price of a European call on a put and a European put on a put?

19.13. Does a lookback call become more valuable or less valuable as we increase the frequency with which we observe the asset price in calculating the minimum?

19.14. Does a down-and-out call become more valuable or less valuable as we increase the frequency with which we observe the asset price in determining whether the barrier has been crossed? What is the answer to the same question for a down-and-in call?

19.15. Explain why a regular European call option is the sum of a down-and-out European call and a down-and-in European call.

19.16. What is the value of a derivative that pays off \$100 in six months if the S&P 500 index is greater than 1,000 and zero otherwise. Assume that the current level of the index is 960, the risk-free rate is 8% per annum, the dividend yield on the index is 3% per annum, and the volatility of the index is 20%.

19.17. Estimate the interest rate paid by P&G on the 5/30 swap in Section 19.3 if (a) the CP rate is 6.5% and the Treasury yield curve is flat at 6% and (b) the CP rate is 7.5% and the Treasury yield curve is flat at 7%.

19.18. Use DerivaGem to calculate the value of:

a. A regular European call option on a non-dividend-paying stock where the stock price is \$50, the strike price is \$50, the risk-free rate is 5% per annum, the volatility is 30%, and the time to maturity is one year.

b. A down-and-out European call which is as in (a) with the barrier at \$45.

c. A down-and-in European call which is as in (a) with the barrier at \$45.

Show that the option in (a) is worth the sum of the values of the options in (b) and (c).

Assignment Questions

19.19. What is the value in dollars of a derivative that pays off £10,000 in one year provided that the dollar–sterling exchange rate is greater than 1.5000 at that time? The current exchange rate is 1.4800. The dollar and sterling interest rates are 4% and 8% per annum, respectively. The volatility of the exchange rate is 12% per annum.

19.20. Consider an up-and-out barrier call option on a non-dividend-paying stock when the stock price is 50, the strike price is 50, the volatility is 30%, the risk-free rate is 5%, the time to maturity is one year, and the barrier is 80. Use DerivaGem to value the option and graph the relationship between (a) the option price and the stock price, (b) the option price and the time to maturity, and (c) the option price and the volatility. Provide an intuitive explanation for the results you get. Show that the delta, theta, and vega for an up-and-out barrier call option can be either positive or negative.

19.21. Suppose that the LIBOR zero rate is flat at 5% with annual compounding. In a five-year swap, company X pays a fixed rate of 6% and receives LIBOR annually on a principal of $100 million. The volatility of the two-year swap rate in three years is 20%.

a. What is the value of the swap?
b. Use DerivaGem to calculate the value of the swap if company X has the option to cancel after three years.
c. Use DerivaGem to calculate the value of the swap if the counterparty has the option to cancel after three years.
d. What is the value of the swap if either side can cancel at the end of three years?