CHAPTER 4 Hedging Strategies Using Futures

Many of the participants in futures markets are hedgers. Their aim is to use futures markets to reduce a particular risk that they face. This risk might relate to the price of oil, a foreign exchange rate, the level of the stock market, or some other variable. A *perfect hedge* is one that completely eliminates the risk. In practice, perfect hedges are rare. To quote one trader: "The only perfect hedge is in a Japanese garden." For the most part, therefore, a study of hedging using futures contracts is a study of the ways in which hedges can be constructed so that they perform as close to perfect as possible.

In this chapter we consider a number of general issues associated with the way hedges are set up. When is a short futures position appropriate? When is a long futures position appropriate? Which futures contract should be used? What is the optimal size of the futures position for reducing risk? At this stage, we restrict our attention to what might be termed *hedge-and-forget* strategies. We assume that no attempt is made to adjust the hedge once it has been put in place. The hedger simply takes a futures position at the beginning of the life of the hedge and closes out the position at the end of the life of the hedge. In Chapter 15 we will examine dynamic hedging strategies in which the hedge is monitored closely and frequent adjustments are made.

Throughout this chapter we will treat futures contracts as forward contract; that is, we will ignore daily settlement. This means that we can ignore the time value of money in most situations because all cash flows occur at the time the hedge is closed out.

4.1 BASIC PRINCIPLES

When an individual or company chooses to use futures markets to hedge a risk, the objective is usually to take a position that neutralizes the risk as far as possible. Consider a company that knows it will gain $10,000 for each 1 cent increase in the price of a commodity over the next three months and lose $10,000 for each 1 cent decrease in the price during the same period. To hedge, the company's treasurer should take a short futures position that is designed to offset this risk. The futures position should lead to a loss of $10,000 for each 1 cent increase in the price of the commodity over the three months and a gain of $10,000 for each 1 cent decrease in the price during this period. If the price of the commodity goes down, the gain on the futures position offsets the loss on the rest of the company's business. If the price of the commodity goes up, the loss on the futures position is offset by the gain on the rest of the company's business.

Short Hedges

A *short hedge* is a hedge, such as the one just described, that involves a short position in futures contracts. A short hedge is appropriate when the hedger already owns an asset and expects to sell it at some time in the future. For example, a short hedge could be used by a farmer who owns some hogs and knows that they will be ready for sale at the local market in two months. A short hedge can also be used when an asset is not owned right now but will be owned at some time in the future. Consider, for example, a U.S. exporter who knows that he or she will receive euros in three months. The exporter will realize a gain if the euro increases in value relative to the U.S. dollar and will sustain a loss if the euro decreases in value relative to the U.S. dollar. A short futures position leads to a loss if the euro increases in value and a gain if it decreases in value. It has the effect of offsetting the exporter's risk.

To provide a more detailed illustration of the operation of a short hedge in a specific situation, we assume that it is May 15 today and that an oil producer has just negotiated a contract to sell 1 million barrels of crude oil. It has been agreed that the price that will apply in the contract is the market price on August 15. The oil producer is therefore in the position where it will gain $10,000 for each 1 cent increase in the price of oil over the next three months and lose $10,000 for each 1 cent decrease in the price during this period. Suppose that the spot price on May 15 is $19 per barrel and the August crude oil futures price on the New York Mercantile Exchange (NYMEX) is $18.75 per barrel. Because each futures contract on NYMEX is for the delivery of 1,000 barrels, the company can hedge its exposure by shorting 1,000 August futures contracts. If the oil producer closes out its position on August 15, the effect of the strategy should be to lock in a price close to $18.75 per barrel.

As an example of what might happen, suppose that the spot price on August 15 proves to be $17.50 per barrel. The company realizes $17.5 million for the oil under its sales contract. Because August is the delivery month for the futures contract, the futures price on August 15 should be very close to the spot price of $17.50 on that date. The company therefore gains approximately

$$\$18.75 - \$17.50 = \$1.25$$

per barrel, or $1.25 million in total from the short futures position. The total amount realized from both the futures position and the sales contract is therefore approximately $18.75 per barrel, or $18.75 million in total.

For an alternative outcome, suppose that the price of oil on August 15 proves to be $19.50 per barrel. The company realizes $19.50 for the oil and loses approximately

$$\$19.50 - \$18.75 = \$0.75$$

per barrel on the short futures position. Again, the total amount realized is approximately $18.75 million. It is easy to see that in all cases the company ends up with approximately $18.75 million. This example is summarized in Table 4.1.

Table 4.1 A short hedge

From the Trader's Desk—May 15

An oil producer has negotiated a contract to sell 1 million barrels of crude oil. The price in the sales contract is the spot price on August 15. Quotes:

Spot price of crude oil: $19.00 per barrel.
August oil futures price: $18.75 per barrel.

Hedging Strategy

May 15: Short 1,000 August futures contracts on crude oil.
August 15: Close out futures position.

Result

The company ensures that it will receive a price close to $18.75 per barrel.

Example 1:

Price of oil on August 15 is $17.50 per barrel.
Company receives $17.50 per barrel under the sales contract.
Company gains about $1.25 per barrel from the futures contract.

Example 2:

Price of oil on August 15 is $19.50 per barrel.
Company receives $19.50 per barrel from the sales contract.
Company loses about $0.75 per barrel from the futures contract.

Long Hedges

Hedges that involve taking a long position in a futures contract are known as *long hedges.* A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

Suppose that it is now January 15. A copper fabricator knows it will require 100,000 pounds of copper on May 15 to meet a certain contract. The spot price of copper is 140 cents per pound, and the May futures price is 120 cents per pound. The fabricator can hedge its position by taking a long position in four May futures contracts on the COMEX division of NYMEX and closing its position on May 15. Each contract is for the delivery of 25,000 pounds of copper. The strategy has the effect of locking in the price of the required copper at close to 120 cents per pound.

This example is summarized in Table 4.2. Suppose that the price of copper on May 15 proves to be 125 cents per pound. Because May is the delivery month for the futures contract, this should be very close to the futures price. The fabricator therefore gains approximately

$$100{,}000 \times (\$1.25 - \$1.20) = \$5{,}000$$

on the futures contracts. It pays $100{,}000 \times \$1.25 = \$125{,}000$ for the copper, making the total cost approximately $\$125{,}000 - \$5{,}000 = \$120{,}000$. For an alternative outcome, suppose that the futures price is 105 cents per pound on May 15. The fabricator then loses approximately

$$100{,}000 \times (\$1.20 - \$1.05) = \$15{,}000$$

on the futures contract and pays $100{,}000 \times \$1.05 = \$105{,}000$ for the copper. Again, the total cost is approximately $120,000, or 120 cents per pound.

Note that it is better for the company to use futures contracts than to buy the copper

Table 4.2 A long hedge

From the Trader's Desk—January 15

A copper fabricator knows it will require 100,000 pounds of copper on May 15 to meet a certain contract. The spot price of copper is 140 cents per pound and the May futures price is 120 cents per pound.

Hedging Strategy

January 15: Take a long position in four May futures contracts on copper.
May 15: Close out the position.

Result

The company ensures that its cost will be close to 120 cents per pound.
Example 1:
Cost of copper on May 15 is 125 cents per pound.
The company gains 5 cents per pound from the futures contract.
Example 2:
Cost of copper on May 15 is 105 cents per pound.
The company loses 15 cents per pound from the futures contract.

on January 15 in the spot market. If it does the latter, it will pay 140 cents per pound instead of 120 cents per pound and will incur both interest costs and storage costs. For a company using copper on a regular basis, this disadvantage would be offset by the convenience yield associated with having the copper on hand. (See Chapter 3 for a discussion of convenience yields.) However, for a company that knows it will not require the copper until May 15, the convenience yield has no value.

Long hedges can also be used to partially offset an existing short position. Consider an investor who has shorted a certain stock. Part of the risk faced by the investor is related to the performance of the stock market as a whole. The investor can neutralize this risk by taking a long position in index futures contracts. This type of hedging strategy is discussed further later in the chapter.

In both the example in Table 4.2 and the example in Table 4.1, we assume that the futures position is closed out in the delivery month. The hedge has the same basic effect if delivery is allowed to happen. However, making or taking delivery can be a costly business. For this reason, delivery is not usually made even when the hedger keeps the futures contract until the delivery month. As will be discussed later, hedgers with long positions usually avoid any possibility of having to take delivery by closing out their positions before the delivery period.

We have also assumed in the two examples that a futures contract is the same as a forward contract. In practice, marking to market does have a small effect on the performance of a hedge. As explained in Chapter 2, it means that the payoff from the futures contract is realized day by day throughout the life of the hedge rather than all at the end.

4.2 ARGUMENTS FOR AND AGAINST HEDGING

The arguments in favor of hedging are so obvious that they hardly need to be stated. Most companies are in the business of manufacturing or retailing or wholesaling, or providing a service. They have no particular skills or expertise in predicting variables such as interest rates, exchange rates, and commodity prices. It makes sense for them to hedge the risks associated with these variables as they arise. The companies can then focus on their main activities—in which presumably they do have particular skills and expertise. By hedging, they avoid unpleasant surprises such as sharp rises in the price of a commodity.

In practice, many risks are left unhedged. In the rest of this section we will explore some of the reasons.

Hedging and Shareholders

One argument sometimes put forward is that the shareholders can, if they wish, do the hedging themselves. They do not need the company to do it for them. This argument is, however, open to question. It assumes that shareholders have as much information about the risks faced by a company as does the company's management. In most instances, this is not the case. The argument also ignores commissions and other transactions costs. These are less expensive per dollar of hedging for large transactions than for small transactions. Hedging is therefore likely to be less expensive when carried out by the company than by individual shareholders. Indeed, the size of futures contracts makes hedging by individual shareholders impossible in many situations.

One thing that shareholders can do far more easily than a corporation is diversify risk. A shareholder with a well-diversified portfolio may be immune to many of the risks faced by a corporation. For example, in addition to holding shares in a company that uses copper, a well-diversified shareholder may hold shares in a copper producer, so that there is very little overall exposure to the price of copper. If companies are acting in the best interests of well-diversified shareholders, it can be argued that hedging is unnecessary in many situations. However, the extent to which managements are in practice influenced by this type of argument is open to question.

Hedging and Competitors

If hedging is not the norm in a certain industry, it may not make sense for one particular company to choose to be different from all others. Competitive pressures within the industry may be such that the prices of the goods and services produced by the industry fluctuate to reflect raw material costs, interest rates, exchange rates, and so on. A company that does not hedge can expect its profit margins to be roughly constant. However, a company that does hedge can expect its profit margins to fluctuate!

To illustrate this point, consider two manufacturers of gold jewelry, SafeandSure Company and TakeaChance Company. We assume that most companies in the industry do not hedge against movements in the price of gold and that TakeaChance Company is no exception. However, SafeandSure Company has decided to be different from its competitors and to use futures contracts to hedge its purchase of gold over the next 18 months. If the price of gold goes up, economic pressures will tend to lead to a corresponding increase in the wholesale price of the jewelry, so that TakeaChance Company's profit margin is unaffected. By contrast, SafeandSure Company's profit margin will increase after the effects of the hedge have been taken into account. If the price of gold goes down, economic pressures will tend to lead to a corresponding decrease in the wholesale price of the jewelry. Again, TakeaChance Company's profit margin is unaffected. However, SafeandSure Company's profit margin goes down. In

Table 4.3 Danger in hedging when competitors do not

Change in gold price	*Effect on price of gold jewelry*	*Effect on profits of TakeaChance Co.*	*Effect on profits of SafeandSure Co.*
Increase	Increase	None	Increase
Decrease	Decrease	None	Decrease

extreme conditions, SafeandSure Company's profit margin could become negative as a result of the "hedging" carried out! This example is summarized in Table 4.3.

This example emphasizes the importance of looking at the big picture when hedging. All the implications of price changes on a company's profitability should be taken into account in the design of a hedging strategy to protect against the price changes.

Other Considerations

It is important to realize that a hedge using futures contracts can result in a decrease or an increase in a company's profits relative to the position it would be in with no hedging. In the example in Table 4.1, if the price of oil goes down, the company loses money on its sale of 1 million barrels of oil, and the futures position leads to an offsetting gain. The treasurer can be congratulated for having had the foresight to put the hedge in place. Clearly, the company is better off than it would be with no hedging. Other executives in the organization, it is hoped, will appreciate the contribution made by the treasurer. If the price of oil goes up, the company gains from its sale of the oil, and the futures position leads to an offsetting loss. The company is in a worse position than it would be with no hedging. Although the hedging decision was perfectly logical, the treasurer may in practice have a difficult time justifying it. Suppose that the price of oil is $21.75 on August 15 in Table 4.1, so that the company loses $3 per barrel on the futures contract. We can imagine a conversation such as the following between the treasurer and the president.

PRESIDENT: This is terrible. We've lost $3 million in the futures market in the space of three months. How could it happen? I want a full explanation.

TREASURER: The purpose of the futures contracts was to hedge our exposure to the price of oil—not to make a profit. Don't forget that we made about $3 million from the favorable effect of the oil price increases on our business.

PRESIDENT: What's that got to do with it? That's like saying that we do not need to worry when our sales are down in California because they are up in New York.

TREASURER: If the price of oil had gone down...

PRESIDENT: I don't care what would have happened if the price of oil had gone down. The fact is that it went up. I really do not know what you were doing playing the futures markets like this. Our shareholders will expect us to have done particularly well this quarter. I'm going to have to explain to them that your actions reduced profits by $3 million. I'm afraid this is going to mean no bonus for you this year.

TREASURER: That's unfair. I was only...

PRESIDENT: Unfair! You are lucky not to be fired. You lost $3 million.

TREASURER: It all depends how you look at it...

It is easy to see why many treasurers are reluctant to hedge! Hedging reduces risk for the company. However, it may increase risks for the treasurer if others do not fully understand what is being done. The only real solution to this problem involves ensuring that all senior executives within the organization fully understand the nature of hedging before a hedging program is put in place. Ideally, hedging strategies are set by a company's board of directors and are clearly communicated to both the company's management and the shareholders.

4.3 BASIS RISK

The hedges in the examples considered so far have been almost too good to be true. The hedger was able to identify the precise date in the future when an asset would be bought or sold. The hedger was then able to use futures contracts to remove almost all the risk arising from the price of the asset on that date. In practice, hedging is often not quite as straightforward. Some of the reasons are as follows:

1. The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract.
2. The hedger may be uncertain as to the exact date when the asset will be bought or sold.
3. The hedge may require the futures contract to be closed out well before its expiration date.

These problems give rise to what is termed *basis risk*. This concept will now be explained.

The Basis

The *basis* in a hedging situation is as follows:[1]

Basis = Spot price of asset to be hedged − Futures price of contract used

If the asset to be hedged and the asset underlying the futures contract are the same, the basis should be zero at the expiration of the futures contract. Prior to expiration, the basis may be positive or negative. From the analysis in Chapter 3, when the underlying asset is a low-interest-rate currency or gold or silver, the futures price is greater than the spot price. This means that the basis is negative. For high-interest-rate currencies and many commodities, the reverse is true, and the basis is positive.

When the spot price increases by more than the futures price, the basis increases. This is referred to as a *strengthening of the basis*. When the futures price increases by more than the spot price, the basis declines. This is referred to as a *weakening of the basis*. Figure 4.1 illustrates how a basis might change over time in a situation where the basis is positive prior to expiration of the futures contract.

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[1] This is the usual definition. However, the alternative definition

Basis = Futures price − Spot price

is sometimes used, particularly when the futures contract is on a financial asset.

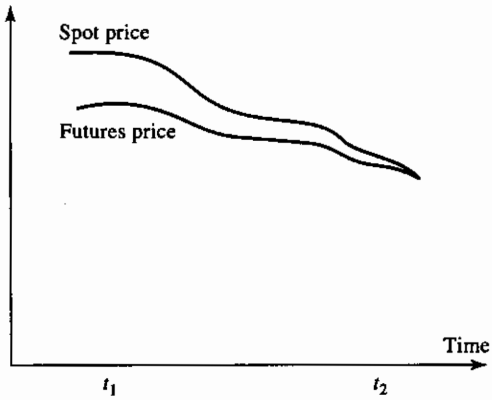

Figure 4.1 Variation of basis over time

To examine the nature of basis risk, we will use the following notation:

S_1: Spot price at time t_1
S_2: Spot price at time t_2
F_1: Futures price at time t_1
F_2: Futures price at time t_2
b_1: Basis at time t_1
b_2: Basis at time t_2

We will assume that a hedge is put in place at time t_1 and closed out at time t_2. As an example, we will consider the case where the spot and futures prices at the time the hedge is initiated are \$2.50 and \$2.20, respectively, and that at the time the hedge is closed out they are \$2.00 and \$1.90, respectively. This means that $S_1 = 2.50$, $F_1 = 2.20$, $S_2 = 2.00$, and $F_2 = 1.90$.

From the definition of the basis,

$$b_1 = S_1 - F_1 \quad \text{and} \quad b_2 = S_2 - F_2$$

and, in our example, $b_1 = 0.30$ and $b_2 = 0.10$.

Consider first the situation of a hedger who knows that the asset will be sold at time t_2 and takes a short futures position at time t_1. The price realized for the asset is S_2 and the profit on the futures position is $F_1 - F_2$. The effective price that is obtained for the asset with hedging is therefore

$$S_2 + F_1 - F_2 = F_1 + b_2$$

In our example, this is \$2.30. The value of F_1 is known at time t_1. If b_2 were also known at this time, a perfect hedge would result. The hedging risk is the uncertainty associated with b_2 and is known as *basis risk*. Consider next a situation where a company knows it will buy the asset at time t_2 and initiates a long hedge at time t_1. The price paid for the asset is S_2 and the loss on the hedge is $F_1 - F_2$. The effective price that is paid with hedging is therefore

$$S_2 + F_1 - F_2 = F_1 + b_2$$

This is the same expression as before and is $2.30 in the example. The value of F_1 is known at time t_1, and the term b_2 represents basis risk.

For investment assets such as currencies, stock indices, gold, and silver, the basis risk tends to be much less than for consumption commodities. The reason, as shown in Chapter 3, is that arbitrage arguments lead to a well-defined relationship between the futures price and the spot price of an investment asset. The basis risk for an investment asset arises mainly from uncertainty as to the level of the risk-free interest rate in the future. In the case of a consumption commodity, imbalances between supply and demand and the difficulties sometimes associated with storing the commodity can lead to large variations in the convenience yield. This in turn leads to a big increase in the basis risk.

The asset that gives rise to the hedger's exposure is sometimes different from the asset underlying the hedge. The basis risk is then usually greater. Define S_2^* as the price of the asset underlying the futures contract at time t_2. As before, S_2 is the price of the asset being hedged at time t_2. By hedging, a company ensures that the price that will be paid (or received) for the asset is

$$S_2 + F_1 - F_2$$

This can be written as

$$F_1 + (S_2^* - F_2) + (S_2 - S_2^*)$$

The terms $S_2^* - F_2$ and $S_2 - S_2^*$ represent the two components of the basis. The $S_2^* - F_2$ term is the basis that would exist if the asset being hedged were the same as the asset underlying the futures contract. The $S_2 - S_2^*$ term is the basis arising from the difference between the two assets.

Note that basis risk can lead to an improvement or a worsening of a hedger's position. Consider a short hedge. If the basis strengthens unexpectedly, the hedger's position improves; if the basis weakens unexpectedly, the hedger's position worsens. For a long hedge, the reverse holds. If the basis strengthens unexpectedly, the hedger's position worsens; if the basis weakens unexpectedly, the hedger's position improves.

Choice of Contract

One key factor affecting basis risk is the choice of the futures contract to be used for hedging. This choice has two components:

1. The choice of the asset underlying the futures contract.
2. The choice of the delivery month.

If the asset being hedged exactly matches an asset underlying a futures contract, the first choice is generally fairly easy. In other circumstances, it is necessary to carry out a careful analysis to determine which of the available futures contracts has futures prices that are most closely correlated with the price of the asset being hedged.

The choice of the delivery month is likely to be influenced by several factors. In the examples earlier in this chapter, we assumed that when the expiration of the hedge corresponds to a delivery month, the contract with that delivery month is chosen. In fact, a contract with a later delivery month is usually chosen in these circumstances. The reason is that futures prices are in some instances quite erratic during the delivery month. Also, a long hedger runs the risk of having to take delivery of the physical asset if the contract is held during the delivery month. Taking delivery can be expensive and inconvenient.

In general, basis risk increases as the time difference between the hedge expiration

Table 4.4 Basis risk in a short hedge

From the Trader's Desk—March 1

It is March 1. A U.S. company expects to receive 50 million Japanese yen at the end of July. The September futures price for the yen is currently 0.7800.

Strategy

The company can

1. Short four September yen futures contracts on March 1.
2. Close out the contract when the yen arrive at the end of July.

Basis Risk

The basis risk arises from the hedger's uncertainty as to the difference between the spot price and September futures price of the Japanese yen at the end of July.

The Outcome

When the yen arrived at the end of July, it turned out that the spot price was 0.7200 and the futures price was 0.7250. It follows that

$$\text{Basis} = 0.7200 - 0.7250 = -0.0050$$
$$\text{Gain on futures} = 0.7800 - 0.7250 = +0.0550$$

The effective price in cents per yen received by the hedger is the end-of-July spot price plus the gain on the futures:

$$0.7200 + 0.0550 = 0.7750$$

This can also be written as the initial September futures price plus the basis:

$$0.7800 - 0.0050 = 0.7750$$

and the delivery month increases. A good rule of thumb is therefore to choose a delivery month that is as close as possible to, but later than, the expiration of the hedge. Suppose delivery months are March, June, September, and December for a particular contract. For hedge expirations in December, January, and February, the March contract will be chosen; for hedge expirations in March, April, and May, the June contract will be chosen; and so on. This rule of thumb assumes that there is sufficient liquidity in all contracts to meet the hedger's requirements. In practice, liquidity tends to be greatest in short maturity futures contracts. The hedger may therefore, in some situations, be inclined to use short maturity contracts and roll them forward. This strategy is discussed later in the chapter.

Illustrations

We now illustrate some of the points made so far in this section. Suppose it is March 1. A U.S. company expects to receive 50 million Japanese yen at the end of July. Yen futures contracts on the Chicago Mercantile Exchange have delivery months of March, June, September, and December. One contract is for the delivery of 12.5 million yen. The criteria mentioned earlier for the choice of a contract suggest that the September contract be chosen for hedging purposes.

The company therefore shorts four September yen futures contracts on March 1. When the yen are received at the end of July, the company closes out its position. The basis risk arises from uncertainty about the difference between the futures price and the

Table 4.5 Basis risk in long hedge

From the Trader's Desk—June 8

It is June 8. A company knows that it will need to purchase 20,000 barrels of crude oil some time in October or November. The current December oil futures price is \$18.00 per barrel.

Strategy

The company

1. Takes a long position in 20 NYM December oil futures contracts on June 8.
2. Closes out the contract when it finds it is ready to purchase the oil.

Basis Risk

The basis risk arises from the hedger's uncertainty as to the difference between the spot price and the December futures price of oil at the time when the oil is required.

The Outcome

The company was ready to purchase the oil on November 10 and closed out its futures contract on that date. The spot price was \$20.00 per barrel, and the futures price was \$19.10 per barrel. It follows that

$$\text{Basis} = \$20.00 - \$19.10 = \$0.90$$

$$\text{Gain on futures} = \$19.10 - \$18.00 = \$1.10$$

The effective cost of the oil purchased is the November 10 price less the gain on the futures:

$$\$20.00 - \$1.10 = \$18.90 \text{ per barrel}$$

This can also be written as the initial December futures price plus the basis:

$$\$18.00 + \$0.90 = \$18.90 \text{ per barrel}$$

spot price at this time. We suppose that the futures price on March 1 in cents per yen is 0.7800 and that the spot and futures prices when the contract is closed out are 0.7200 and 0.7250, respectively. The basis is −0.0050, and the gain from the futures contracts is 0.0550. The effective price obtained in cents per yen is the spot price plus the gain on the futures:

$$0.7200 + 0.0550 = 0.7750$$

This can also be written as the initial futures price plus the basis:

$$0.7800 - 0.0050 = 0.7750$$

The company receives a total of 50×0.00775 million dollars, or \$387,500. This example is summarized in Table 4.4.

For our next example, we suppose it is June 8, and a company knows that it will need to purchase 20,000 barrels of crude oil at some time in October or November. Oil futures contracts are currently traded for delivery every month on NYMEX, and the contract size is 1,000 barrels. Following the criteria indicated, the company decides to use the December contract for hedging. On June 8 it takes a long position in 20 December contracts. At that time, the futures price is \$18.00 per barrel. The company finds that it is ready to purchase the crude oil on November 10. It therefore closes out its futures contract on that date. The basis risk arises from uncertainty as to what the basis will be on the day the contract is closed out. We suppose that the spot price and futures price on November 10 are \$20.00 per barrel and \$19.10 per barrel, respectively. The basis is therefore \$0.90, and the effective price paid is \$18.90 per barrel, or \$378,000 in total. This example is summarized in Table 4.5.

4.4 MINIMUM VARIANCE HEDGE RATIO

The *hedge ratio* is the ratio of the size of the position taken in futures contracts to the size of the exposure. Up to now we have always used a hedge ratio of 1.0. In Table 4.5, for example, the hedger's exposure was on 20,000 barrels of oil, and futures contracts were entered into for the delivery of exactly this amount of oil. If the objective of the hedger is to minimize risk, setting the hedge ratio equal to 1.0 is not necessarily optimal.

We will use the following notation:

- δS: Change in spot price, S, during a period of time equal to the life of the hedge
- δF: Change in futures price, F, during a period of time equal to the life of the hedge
- σ_S: Standard deviation of δS
- σ_F: Standard deviation of δF
- ρ: Coefficient of correlation between δS and δF
- h^*: Hedge ratio that minimizes the variance of the hedger's position

In the Appendix at the end of the chapter we show that

$$h^* = \rho \frac{\sigma_S}{\sigma_F} \tag{4.1}$$

The optimal hedge ratio is the product of the coefficient of correlation between δS and δF and the ratio of the standard deviation of δS to the standard deviation of δF. Figure 4.2 shows how the variance of the value of the hedger's position depends on the hedge ratio chosen.

If $\rho = 1$ and $\sigma_F = \sigma_S$, the hedge ratio, h^*, is 1.0. This result is to be expected, because in this case the futures price mirrors the spot price perfectly. If $\rho = 1$ and $\sigma_F = 2\sigma_S$, the

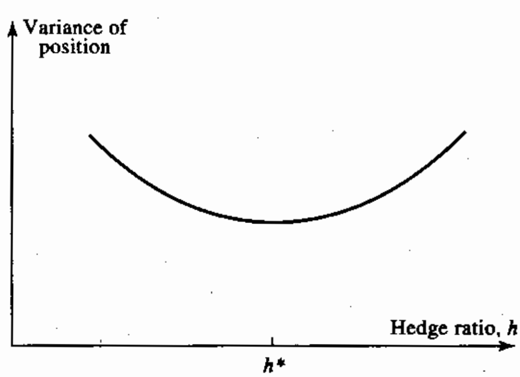

Figure 4.2 Dependence of variance of hedger's position on hedge ratio

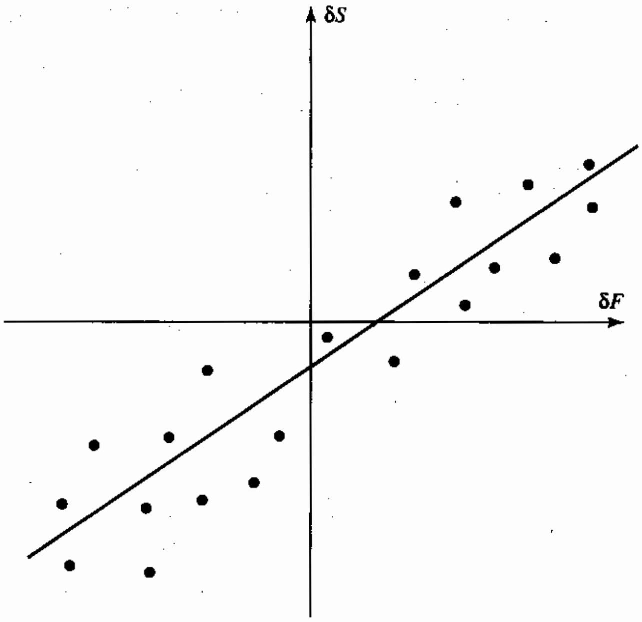

Figure 4.3 Regression of change in spot price against change in futures price

hedge ratio h^* is 0.5. This result is also as expected, because in this case the futures price always changes by twice as much as the spot price.

The optimal hedge ratio, h^*, is the slope of the best fit line when δS is regressed against δF, as indicated in Figure 4.3. This is intuitively reasonable, because we require h^* to correspond to the ratio of changes in δS to changes in δF. The *hedge effectiveness* can be defined as the proportion of the variance that is eliminated by hedging. This is ρ^2, or

$$h^{*2}\frac{\sigma_F^2}{\sigma_S^2}$$

The parameters ρ, σ_F, and σ_S in equation (4.1) are usually estimated from historical data on δS and δF. (The implicit assumption is that the future will in some sense be like the past.) A number of equal nonoverlapping time intervals are chosen, and the values of δS and δF for each of the intervals are observed. Ideally, the length of each time interval is the same as the length of the time interval for which the hedge is in effect. In practice, this sometimes severely limits the number of observations that are available, and a shorter time interval is used.

Optimal Number of Contracts

Define variables as follows:

- N_A: Size of position being hedged (units)
- Q_F: Size of one futures contract (units)
- N^*: Optimal number of futures contracts for hedging

The futures contracts used should have a face value of h^*N_A. The number of futures

Table 4.6 Data to calculate minimum variance hedge ratio when heating oil futures contract is used to hedge purchase of jet fuel

Month i	*Change in futures price per gallon* ($= x_i$)	*Change in fuel price per gallon* ($= y_i$)
1	0.021	0.029
2	0.035	0.020
3	−0.046	−0.044
4	0.001	0.008
5	0.044	0.026
6	−0.029	−0.019
7	−0.026	−0.010
8	−0.029	−0.007
9	0.048	0.043
10	−0.006	0.011
11	−0.036	−0.036
12	−0.011	−0.018
13	0.019	0.009
14	−0.027	−0.032
15	0.029	0.023

contracts required is therefore given by

$$N^* = \frac{h^* N_A}{Q_F} \tag{4.2}$$

Example

An airline expects to purchase two million gallons of jet fuel in one month and decides to use heating oil futures for hedging. (The article by Nikkhah referenced at the end of the chapter discusses this type of strategy.) We suppose that Table 4.6 gives, for 15 successive months, data on the change, δS, in the jet fuel price per gallon and the corresponding change, δF, in the futures price for the contract on heating oil that would be used for hedging price changes during the month. The number of observations, which we will denote by n, is 15. We will denote the ith observations on δF and δS by x_i and y_i, respectively. From Table 4.6,

$$\sum x_i = -0.013 \qquad \sum x_i^2 = 0.0138$$

$$\sum y_i = 0.003 \qquad \sum y_i^2 = 0.0097$$

$$\sum x_i y_i = 0.0107$$

Standard formulas from statistics give the estimate of σ_F as

$$\sqrt{\frac{\sum x_i^2}{n-1} - \frac{(\sum x_i)^2}{n(n-1)}} = 0.0313$$

The estimate of σ_S is

$$\sqrt{\frac{\sum y_i^2}{n-1} - \frac{(\sum y_i)^2}{n(n-1)}} = 0.0263$$

The estimate of ρ is

$$\frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{[n\sum x_i^2 - (\sum x_i)^2][n\sum y_i^2 - (\sum y_i)^2]}} = 0.928$$

From equation (4.1), the minimum variance hedge ratio, h^*, is therefore

$$0.928 \times \frac{0.0263}{0.0313} = 0.78$$

Each heating oil contract traded on NYMEX is on 42,000 gallons of heating oil. From equation (4.2), the optimal number of contracts is

$$\frac{0.78 \times 2{,}000{,}000}{42{,}000} = 37.14$$

or, rounding to the nearest whole number, 37.

4.5 STOCK INDEX FUTURES

Stock index futures can be used to hedge an equity portfolio. Define:

P: Current value of the portfolio

A: Current value of the stocks underlying one futures contract

If the portfolio mirrors the index, a hedge ratio of 1 is clearly appropriate, and equation (4.2) shows that the number of futures contracts that should be shorted is

$$N^* = \frac{P}{A} \qquad \textbf{(4.3)}$$

Suppose, for example, that a portfolio worth \$1 million mirrors the S&P 500. The current value of the index is 1,000, and each futures contract is on \$250 times the index. In this case $P = 1{,}000{,}000$ and $A = 250{,}000$, so that four contracts should be shorted to hedge the portfolio.

When the portfolio does not exactly mirror the index, we can use the parameter beta (β) from the capital asset pricing model to determine the appropriate hedge ratio. Beta is the slope of the best fit line obtained when excess return on the portfolio over the risk-free rate is regressed against the excess return of the market over the risk-free rate. When $\beta = 1.0$, the return on the portfolio tends to mirror the return on the market; when $\beta = 2.0$, the excess return on the portfolio tends to be twice as great as the excess return on the market; when $\beta = 0.5$, it tends to be half as great; and so on.

Assuming that the index underlying the futures contract is a proxy for the market, it can be shown that the appropriate hedge ratio is the beta of the portfolio. From equation (4.2), this means that

$$N^* = \beta\frac{P}{A} \qquad \textbf{(4.4)}$$

This formula assumes that the maturity of the futures contract is close to the maturity of the hedge and ignores the daily settlement of the futures contract.[2]

We illustrate that this formula gives good results with an example. Suppose that

$$\text{Value of S \& P 500 index} = 1{,}000$$
$$\text{Value of portfolio} = \$5{,}000{,}000$$
$$\text{Risk-free interest rate} = 10\% \text{ per annum}$$
$$\text{Dividend yield on index} = 4\% \text{ per annum}$$
$$\text{Beta of portfolio} = 1.5$$

We assume that a futures contract on the S&P 500 with four months to maturity is used to hedge the value of the portfolio over the next three months. One futures contract is for delivery of \$250 times the index. From equation (3.12), the current futures price should be

$$1{,}000e^{(0.10-0.04)\times 4/12} = 1{,}020.20$$

From equation (4.4), the number of futures contracts that should be shorted to hedge the portfolio is

$$1.5 \times \frac{5{,}000{,}000}{250{,}000} = 30$$

Suppose the index turns out to be 900 in three months. The futures price will be

$$900e^{(0.10-0.04)\times 1/12} = 904.51$$

The gain from the short futures position is therefore

$$30 \times (1020.20 - 904.51) \times 250 = \$867,$$

The loss on the index is 10%. The index pays a dividend of 4% per annum, or 1% per three months. When dividends are taken into account, an investor in the index would therefore earn −9% in the three-month period. The risk-free interest rate is approximately 2.5% per three months.[3] Because the portfolio has a β of 1.5,

$$\text{Expected return on portfolio} - \text{Risk-free interest rate}$$
$$= 1.5 \times (\text{Return on index} - \text{Risk-free interest rate})$$

It follows that the expected return (%) on the portfolio is

$$2.5 + [1.5 \times (-9.0 - 2.5)] = -14.75$$

[2] A small adjustment known as *tailing the hedge* can be used to take account of the daily settlement when a futures contract is used for hedging. For a discussion of this, see D. Duffie, *Futures Markets*, Upper Saddle River, NJ: Prentice Hall, 1989; R. Rendleman, "A Reconciliation of Potentially Conflicting Approaches to Hedging with Futures," *Advances in Futures and Options Research* 6 (1993). Problem 4.20 deals with this issue.

[3] For ease of presentation, the fact that the interest rate and dividend yield are continuously compounded has been ignored. This makes very little difference.

Table 4.7 Performance of stock index hedge

Value of index in three months	900.00	950.00	1,000.00	1,050.00	1,100.00
Futures price of index in three months	904.51	954.76	1,005.01	1,055.26	1,105.51
Gain (loss) on futures position ($000)	867,676	490,796	113,916	(262,964)	(639,843)
Value of portfolio (including dividends) in three months ($000)	4,262,500	4,637,500	5,012,500	5,387,500	5,762,500
Total value of position in three months ($000)	5,130,176	5,128,296	5,126,416	5,124,537	5,122,657

The expected value of the portfolio (inclusive of dividends) at the end of the three months is therefore

$$\$5,000,000 \times (1 - 0.1475) = \$4,262,500$$

It follows that the expected value of the hedger's position, including the gain on the hedge, is

$$\$4,262,500 + \$867,676 = \$5,130,176$$

Table 4.7 summarizes these calculations together with similar calculations for other values of the index at maturity. It can be seen that the total value of the hedger's position in three months is almost independent of the value of the index.

Table 4.7 assumes that the dividend yield on the index is predictable, the risk-free interest rate remains constant, and the return on the index over the three-month period is perfectly correlated with the return on the portfolio. In practice, these assumptions do not hold perfectly, and the hedge works rather less well than is indicated by Table 4.7.

Reasons for Hedging an Equity Portfolio

Table 4.7 shows that the hedging scheme results in a value for the hedger's position close to $5,125,000 at the end of three months. This is greater than the $5,000,000 initial value of the position by about 2.5%. There is no surprise here. The risk-free interest rate is 10% per annum, or about 2.5% per quarter. The hedge results in the hedger's position growing at the risk-free interest rate.

It is natural to ask why the hedger should go to the trouble of using futures contracts. To earn the risk-free interest rate, the hedger can simply sell the portfolio and invest the proceeds in Treasury bills.

One answer to this question is that hedging can be justified if the hedger feels that the stocks in the portfolio have been chosen well. In these circumstances, the hedger might be very uncertain about the performance of the market as a whole, but confident that the stocks in the portfolio will outperform the market (after appropriate adjustments have been made for the beta of the portfolio). A hedge using index futures removes the risk arising from market moves and leaves the hedger exposed only to the performance of the portfolio relative to the market. Another reason for hedging may be that the hedger is planning to hold a portfolio for a long period of time and requires short-term protection in an uncertain market situation. The alternative strategy of selling the portfolio and buying it back later might involve unacceptably high transaction costs.

Changing Beta

In the example in Table 4.7, the beta of the hedger's portfolio is reduced to zero. Sometimes futures contracts are used to change the beta of a portfolio to some value other than zero. In the example, to reduce the beta of the portfolio from 1.5 to 0.75, the number of contracts shorted should be 15 rather than 30; to increase the beta of the portfolio to 2.0, a long position in 10 contracts should be taken; and so on. In general, to change the beta of the portfolio from β to β^*, where $\beta > \beta^*$, a short position in

$$(\beta - \beta^*)\frac{P}{A}$$

contracts is required. When $\beta < \beta^*$, a long position in

$$(\beta^* - \beta)\frac{P}{A}$$

contracts is required.

Exposure to the Price of an Individual Stock

Some exchanges do trade futures contracts on selected individual stocks, but in most cases a position in an individual stock can only be hedged using a stock index futures contact.

Hedging an exposure to the price of an individual stock using index futures contracts is similar to hedging a stock portfolio. The number of index futures contracts that the hedger should short into is given by $\beta P/A$, where β is the beta of the stock, P is the total value of the shares owned, and A is the current value of the stocks underlying one index futures contract. Note that although the number of contracts entered into is calculated in the same way as it is when a portfolio of stocks is being hedged, the performance of the hedge is considerably worse. The hedge provides protection only against the risk arising from market movements, and this risk is a relatively small proportion of the total risk in the price movements of individual stocks. The hedge is appropriate when an investor feels that the stock will outperform the market but is unsure about the performance of the market. It can also be used by an investment bank that has underwritten a new issue of the stock and wants protection against moves in the market as a whole.

Consider an investor who in June holds 20,000 IBM shares, each worth \$100. The investor feels that the market will be very volatile over the next month but that IBM has a good chance of outperforming the market. The investor decides to use the August futures contract on the S&P 500 to hedge the position during the one-month period. The β of IBM is estimated at 1.1. The current level of the index is 900, and the current futures price for the August contract on the S&P 500 is 908. Each contract is for delivery of \$250 times the index. In this case $P = 20{,}000 \times 100 = 2{,}000{,}000$ and $A = 900 \times 250 = 225{,}000$. The number of contracts that should be shorted is therefore

$$1.1 \times \frac{2{,}000{,}000}{225{,}000} = 9.78$$

Rounding to the nearest integer, the hedger shorts 10 contracts, closing out the position

Table 4.8 Hedging a position in an individual stock

From the Trader's Desk—June

An investor holds 20,000 IBM shares. The investor is concerned about the volatility of the market during the next month. The current market price of IBM is \$100, the current level on the S&P 500 is 900, and the August futures price of the S&P 500 is 908.

Strategy

The investor

1. Shorts 10 August futures contracts on the S&P 500.
2. Closes out the position one month later.

Outcome

One month later the price of IBM is \$125 and the August futures price of the S&P 500 is 1080. The investor gains

$$20,000 \times (\$125 - \$100) = \$500{,}000$$

on the IBM holding and loses

$$10 \times 250 \times (1080 - 908) = \$430{,}000$$

on the futures position.

one month later. Suppose IBM rises to \$125 during the month, and the futures price of the S&P 500 rises to 1080. The investor gains $20{,}000 \times (\$125 - \$100) = \$500{,}000$ on IBM while losing $10 \times 250 \times (1080 - 908) = \$430{,}000$ on the futures contracts. The example is summarized in Table 4.8.

In this example, the hedge offsets a gain on the underlying asset with a loss on the futures contracts. The offset might seem to be counterproductive. However, it cannot be emphasized often enough that the purpose of a hedge is to reduce risk. A hedge tends to make unfavorable outcomes less unfavorable and favorable outcomes less favorable.

4.6 ROLLING THE HEDGE FORWARD

Sometimes the expiration date of the hedge is later than the delivery dates of all the futures contracts that can be used. The hedger must then roll the hedge forward by closing out one futures contract and taking the same position in a futures contract with a later delivery date. Hedges can be rolled forward many times. Consider a company that wishes to use a short hedge to reduce the risk associated with the price to be received for an asset at time T. If there are futures contracts 1, 2, 3, ..., n (not all necessarily in existence at the present time) with progressively later delivery dates, the company can use the following strategy:

Time t_1: Short futures contract 1.

Time t_2: Close out futures contract 1.
Short futures contract 2.

Time t_3: Close out futures contract 2.
Short futures contract 3.

⋮

Table 4.9 Rolling the hedge forward

From the Trader's Desk—April 2001

The price of oil is $19 per barrel. A company knows it will have 100,000 barrels of oil to sell in June 2002 and wishes to hedge its position. Oil futures contracts are traded on the NYMEX exchange for every delivery month up to one year in the future. However, only the first six delivery months provide sufficient liquidity to meet the company's needs. The contract size is 1,000 barrels.

Strategy

Apr. 2001: The company shorts 100 October 2001 contracts.

Sept. 2001: The company closes out the 100 October 2001 contracts. The company shorts 100 March 2002 contracts.

Feb. 2002: The company closes out the 100 March 2002 contracts. The company shorts 100 July 2002 contracts.

June 2002: The company closes out the 100 July contracts. The company sells 100,000 barrels of oil.

The Outcome

Oct. 2001 futures contract: Shorted in April 2001 at $18.20 and closed out in September 2001 at $17.40.

Sept. 2001 futures contract: Shorted in September 2001 at $17.00 and closed out in February 2002 at $16.50.

July 2002 futures contract: Shorted in February 2002 at $16.30 and closed out in June 2002 at $15.90.

Spot oil price in June 2002: $16 per barrel.

The gain from the futures contract, ignoring the time value of money, is

$$(\$18.20 - \$17.40) + (\$17.00 - \$16.50) + (\$16.30 - \$15.90) = \$1.70 \text{ per barrel}$$

This partly offsets the $3 decline in oil prices between April 2001 and June 2002.

Time t_n: Close out futures contract $n - 1$.
Short futures contract n.

Time T: Close out futures contract n.

An example of this strategy is shown in Table 4.9. In April 2001 a company realizes that it will have 100,000 barrels of oil to sell in June 2002 and decides to hedge its risk with a hedge ratio of 1.0. The current spot price is $19. Although futures contracts are traded with maturities stretching several years into the future, we suppose that only the first six delivery months have sufficient liquidity to meet the company's needs. The company therefore shorts 100 October 2001 contracts. In September 2001 it rolls the hedge forward into the March 2002 contract. In February 2002 it rolls the hedge forward again into the July 2002 contract.

As one possible outcome, we suppose that the price of oil drops $3 to $16 per barrel in June 2002. We suppose that the October 2001 futures contract was shorted at $18.20 per barrel and closed out at $17.40 per barrel for a profit of $0.80 per barrel; the March 2002 contract was shorted at $17.00 per barrel and closed out at $16.50 per barrel for a profit of $0.50 per barrel. The July 2002 contract was shorted at $16.30 per barrel and closed out at $15.90 per barrel for a profit of $0.40 per barrel. In this case, ignoring the time value of money, the futures contracts provide a total of $1.70 per barrel compensation for the $3 per barrel oil price decline.

Receiving only $1.70 per barrel compensation for a price decline of $3.00 may appear unsatisfactory. However, we cannot expect total compensation for a price decline when futures prices are below spot prices. The best we can hope for is to lock in the futures price that would apply to a June 2002 contract if it were actively traded.

Metallgesellschaft

Sometimes rolling the hedge forward can lead to cash flow pressures. The problem was illustrated dramatically by the activities of a German company, Metallgesellschaft (MG), in the early 1990s.

MG sold a huge volume of 5- to 10-year heating oil and gasoline fixed-price supply contracts to its customers at 6 to 8 cents above market prices. It hedged its exposure with long positions in short-dated futures contracts that were rolled over. As it turned out, the price of oil fell and there were margin calls on the futures position. Considerable short-term cash flow pressures were placed on MG. The members of MG who devised the hedging strategy argued that these short-term cash outflows were offset by positive cash flows that would ultimately be realized on the long-term fixed-price contracts. However, the company's senior management and its bankers became concerned about the huge cash drain. As a result, the company closed out all the hedge positions and agreed with its customers that the fixed-price contracts would be abandoned. The result was a loss to MG of $1.33 billion.[4]

4.7 SUMMARY

This chapter has discussed various ways in which a company can take a position in futures contracts to offset an exposure to the price of an asset. If the exposure is such that the company gains when the price of the asset increases and loses when the price of the asset decreases, a short hedge is appropriate. If the exposure is the other way round (i.e., the company gains when the price of the asset decreases and loses when the price of the asset increases), a long hedge is appropriate.

Hedging is a way of reducing risk. As such, it should be welcomed by most executives. In reality, there are a number of theoretical and practical reasons that companies do not hedge. On a theoretical level, we can argue that shareholders, by holding well-diversified portfolios, can eliminate many of the risks faced by a company. They do not require the company to hedge these risks. On a practical level, a company may find that it is increasing rather than decreasing risk by hedging if none of its competitors does so. Also, a treasurer may fear criticism from other executives if the company makes a gain from movements in the price of the underlying asset and a loss on the hedge.

An important concept in hedging is basis risk. The basis is the difference between the spot price of an asset and its futures price. Basis risk is created by a hedger's uncertainty as to what the basis will be at maturity of the hedge. Basis risk is generally greater for consumption assets than for investment assets.

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[4] For a discussion of MG, see "MG's Trial by Essay," *Risk* (October 1994): 228–34, and M. Miller and C. Culp, "Risk Management Lessons from Metallgesellschaft," *Journal of Applied Corporate Finance* 7(4) (Winter 1995): 62–76.

The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure. It is not always optimal to use a hedge ratio of 1.0. If the hedger wishes to minimize the variance of a position, a hedge ratio different from 1.0 may be appropriate. The optimal hedge ratio is the slope of the best fit line obtained when changes in the spot price are regressed against changes in the futures price.

Stock index futures can be used to hedge the systematic risk in an equity portfolio. The number of futures contracts required is the beta of the portfolio multiplied by the ratio of the value of the portfolio to the value of one futures contract. Stock index futures can also be used to change the beta of a portfolio without changing the stocks comprising the portfolio.

When there is no liquid futures contract that matures later than the expiration of the hedge, a strategy known as rolling the hedge forward may be appropriate. This involves entering into a sequence of futures contracts. When the first futures contract is near expiration, it is closed out and the hedger enters into a second contract with a later delivery month. When the second contract is close to expiration, it is closed out and the hedger enters into a third contract with a later delivery month; and so on. Rolling the hedge forward works well if there is a close correlation between changes in the futures prices and changes in the spot prices.

Suggestions for Further Reading

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Franckle, C. T. "The Hedging Performance of the New Futures Market: Comment." *Journal of Finance* 35 (December 1980): 1273–79.

Johnson, L. L. "The Theory of Hedging and Speculation in Commodity Futures Markets." *Review of Economics Studies* 27 (October 1960): 139–51.

McCabe, G. M., and C. T. Franckle. "The Effectiveness of Rolling the Hedge Forward in the Treasury Bill Futures Market." *Financial Management* 12 (Summer 1983): 21–9.

Miller, M., and C. Culp. "Risk Management Lessons from Metallgesellschaft." *Journal of Applied Corporate Finance* 7(4) (Winter 1995): 62–76.

Nikkhah, S. "How End Users Can Hedge Fuel Costs in Energy Markets." *Futures* (October 1987): 66–7.

Stulz, R. M. "Optimal Hedging Policies." *Journal of Financial and Quantitative Analysis* 19 (June 1984): 127–40.

Quiz (Answers at End of Book)

4.1. Under what circumstances are (a) a short hedge and (b) a long hedge appropriate?

4.2. Explain what is meant by *basis risk* when futures contracts are used for hedging.

4.3. Explain what is meant by a *perfect hedge*. Does a perfect hedge always lead to a better outcome than an imperfect hedge? Explain your answer.

4.4. Under what circumstances does a minimum variance hedge portfolio lead to no hedging at all?

4.5. Give three reasons that the treasurer of a company might not hedge the company's exposure to a particular risk.

4.6. Suppose that the standard deviation of quarterly changes in the prices of a commodity is \$0.65, the standard deviation of quarterly changes in a futures price on the commodity is \$0.81, and the coefficient of correlation between the two changes is 0.8. What is the optimal hedge ratio for a three-month contract? What does it mean?

4.7. A company has a \$20 million portfolio with a beta of 1.2. It would like to use futures contracts on the S&P 500 to hedge its risk. The index is currently standing at 1080, and each contract is for delivery of \$250 times the index. What is the hedge that minimizes risk? What should the company do if it wants to reduce the beta of the portfolio to 0.6?

Questions and Problems (Answers in Solutions Manual)

4.8. In the Chicago Board of Trade's corn futures contract, the following delivery months are available: March, May, July, September, and December. State the contract that should be used for hedging when the expiration of the hedge is in
a. June
b. July
c. January

4.9. Does a perfect hedge always succeed in locking in the current spot price of an asset for a future transaction? Explain your answer.

4.10. Explain why a short hedger's position improves when the basis strengthens unexpectedly and worsens when the basis weakens unexpectedly.

4.11. Imagine you are the treasurer of a Japanese company exporting electronic equipment to the United States. Discuss how you would design a foreign exchange hedging strategy and the arguments you would use to sell the strategy to your fellow executives.

4.12. Suppose that in Table 4.5 the company decides to use a hedge ratio of 0.8. How does the decision affect the way in which the hedge is implemented and the result?

4.13. "If the minimum variance hedge ratio is calculated as 1.0, the hedge must be perfect." Is this statement true? Explain your answer.

4.14. "If there is no basis risk, the minimum variance hedge ratio is always 1.0." Is this statement true? Explain your answer.

4.15. "When the convenience yield is high, long hedges are likely to be particularly attractive." Explain this statement. Illustrate it with an example.

4.16. The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?

4.17. A corn farmer argues, "I do not use futures contracts for hedging. My real risk is not the price of corn. It is that my whole crop gets wiped out by the weather." Discuss this viewpoint. Should the farmer estimate his or her expected production of corn and hedge to try to lock in a price for expected production?

4.18. On July 1, an investor holds 50,000 shares of a certain stock. The market price is \$30 per share. The investor is interested in hedging against movements in the market over the next month and decides to use the September Mini S&P 500 futures contract. The index is currently 1,500 and one contract is for delivery of \$50 times the index. The beta of the stock is 1.3. What strategy should the investor follow?

4.19. Suppose that in Table 4.9 the company decides to use a hedge ratio of 1.5. How does the decision affect the way the hedge is implemented and the result?

4.20. A U.S. company is interested in using the futures contracts traded on the CME to hedge its Australian dollar exposure. Define r as the interest rate (all maturities) on the U.S. dollar and r_f as the interest rate (all maturities) on the Australian dollar. Assume that r and r_f are constant and that the company uses a contract expiring at time T to hedge an exposure at time t ($T > t$).

a. Using the results in Chapter 3, show that the optimal hedge ratio is

$$e^{(r_f - r)(T-t)}$$

b. Show that, when t is one day, the optimal hedge ratio is almost exactly S_0/F_0, where S_0 is the current spot price of the currency and F_0 is the current futures price of the currency for the contract maturing at time T.

c. Show that the company can take account of the daily settlement of futures contracts for a hedge that lasts longer than one day by adjusting the hedge ratio so that it always equals the spot price of the currency divided by the futures price of the currency.

Assignment Questions

4.21. The following table gives data on monthly changes in the spot price and the futures price for a certain commodity. Use the data to calculate a minimum variance hedge ratio.

Spot price change	+0.50	+0.61	−0.22	−0.35	+0.79
Futures price change	+0.56	+0.63	−0.12	−0.44	+0.60
Spot price change	+0.04	+0.15	+0.70	−0.51	−0.41
Futures price change	−0.06	+0.01	+0.80	−0.56	−0.46

4.22. It is July 16. A company has a portfolio of stocks worth \$100 million. The beta of the portfolio is 1.2. The company would like to use the CME December futures contract on the S&P 500 to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The index is currently 1,000, and each contract is on \$250 times the index.

a. What position should the company take?

b. Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?

4.23. It is now October 2001. A company anticipates that it will purchase 1 million pounds of copper in each of February 2002, August 2002, February 2003, and August 2003. The company has decided to use the futures contracts traded in the COMEX division of the New York Mercantile Exchange to hedge its risk. One contract is for the delivery of 25,000 pounds of copper. The initial margin is \$2,000 per contract and the maintenance margin is \$1,500 per contract. The company's policy is to hedge 80% of its exposure.

Contracts with maturities up to 13 months into the future are considered to have sufficient liquidity to meet the company's needs. Devise a hedging strategy for the company.

Assume the market prices (in cents per pound) today and at future dates are as follows. What is the impact of the strategy you propose on the price the company pays for copper? What is the initial margin requirement in October 2001? Is the company subject to any margin calls?

Date	*Oct. 2001*	*Feb. 2002*	*Aug. 2002*	*Feb. 2003*	*Aug. 2003*
Spot price	72.00	69.00	65.00	77.00	88.00
Mar. 2000 futures price	72.30	69.10			
Sept. 2000 futures price	72.80	70.20	64.80		
Mar. 2001 futures price		70.70	64.30	76.70	
Sept. 2001 futures price			64.20	76.50	88.20

4.24. A fund manager has a portfolio worth \$50 million with a beta of 0.87. The manager is concerned about the performance of the market over the next two months and plans to use three-month futures contracts on the S&P 500 to hedge the risk. The current level of the index is 1,250, one contract is on 250 times the index, the risk-free rate is 6% per annum, and the dividend yield on the index is 3% per annum.

a. What is the theoretical futures price for the three-month futures contract?
b. What position should the fund manager take to eliminate all exposure to the market over the next two months?
c. Calculate the effect of your strategy on the fund manager's returns if the level of the market in two months is 1,000, 1,100, 1,200, 1,300, and 1,400.

APPENDIX

Proof of the Minimum Variance Hedge Ratio Formula

Suppose we expect to sell N_A units of an asset at time t_2 and choose to hedge at time t_1 by shorting futures contracts on N_F units of a similar asset. The hedge ratio, which we will denote by h, is

$$h = \frac{N_F}{N_A} \tag{4A.1}$$

We will denote the total amount realized for the asset when the profit or loss on the hedge is taken into account by Y, so that

$$Y = S_2 N_A - (F_2 - F_1) N_F$$

or

$$Y = S_1 N_A + (S_2 - S_1) N_A - (F_2 - F_1) N_F \tag{4A.2}$$

where S_1 and S_2 are the asset prices at times t_1 and t_2, and F_1 and F_2 are the futures prices at times t_1 and t_2. From equation (4A.1), the expression for Y in equation (4A.2) can be written

$$Y = S_1 N_A + N_A(\delta S - h\delta F) \tag{4A.3}$$

where

$$\delta S = S_2 - S_1$$

$$\delta F = F_2 - F_1$$

Because S_1 and N_A are known at time t_1, the variance of Y in equation (4A.3) is minimized when the variance of $\delta S - h\delta F$ is minimized. The variance of $\delta S - h\delta F$ equals

$$\sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F$$

This can be written as

$$(h\sigma_F - \rho\sigma_S)^2 + \sigma_S^2 - \rho^2\sigma_S^2$$

The second and third terms do not involve h. The variance is therefore minimized when

$$(h\sigma_F - \rho\sigma_S)^2$$

is zero; that is, when

$$h = \rho\frac{\sigma_S}{\sigma_F}$$