CHAPTER 6 Swaps

A swap is an agreement between two companies to exchange cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated. Usually the calculation of the cash flows involves the future values of one or more market variables.

A forward contract can be viewed as simple example of a swap. Suppose it is March 1, 2001, and a company enters into a forward contract to buy 100 ounces of gold for \$300 per ounce in one year. The company can sell the gold in one year as soon as it is received. The forward contract is therefore equivalent to a swap where the company agrees that on March 1, 2002, it will pay \$30,000 and receive $100S$, where S is the market price of one ounce of gold on that date.

Whereas a forward contract leads to the exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges taking place on several future dates. The first swap contracts were negotiated in the early 1980s. Since then the market has seen phenomenal growth. In this chapter we examine how swaps are designed, how they are used, and how they can be valued. Our discussion centers around the two popular types of swaps: plain vanilla interest rate swaps and fixed-for-fixed currency swaps. Other types of swaps are discussed in Chapter 19.

6.1 MECHANICS OF INTEREST RATE SWAPS

The most common type of swap is a "plain vanilla" interest rate swap. In this a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time.

The floating rate in many interest rate swap agreements is the London Interbank Offer Rate (LIBOR). This was introduced in Chapter 5. LIBOR is the rate of interest offered by banks on deposits from other banks in Eurocurrency markets. One-month LIBOR is the rate offered on one-month deposits, three-month LIBOR is the rate offered on three-month deposits, and so on. LIBOR rates are determined by trading between banks and change frequently so that the supply of funds in the interbank market equals the demand for funds in that market. Just as prime is often the reference rate of interest for floating-rate loans in the domestic financial market, LIBOR is a reference rate of interest for loans in international financial markets. To understand how it is used, consider a five-year loan with a rate of interest specified as six-month LIBOR plus 0.5% per annum. The life of the loan is divided into ten periods each six months in length. For each period the rate of interest is set at 0.5% per annum above the six-month LIBOR rate at the beginning of the period. Interest is paid at the end of the period.

Illustration

Consider a hypothetical three-year swap initiated on March 5, 2001, between Microsoft and Intel. We suppose Microsoft agrees to pay to Intel an interest rate of 5% per annum on a notional principal of $100 million, and in return Intel agrees to pay Microsoft the six-month LIBOR rate on the same notional principal. We assume the agreement specifies that payments are to be exchanged every six months, and the 5% interest rate is quoted with semiannual compounding. This swap is represented diagrammatically in Figure 6.1.

The first exchange of payments would take place on September 5, 2001, six months after the initiation of the agreement. Microsoft would pay Intel $2.5 million. This is the interest on the $100 million principal for six months at 5%. Intel would pay Microsoft interest on the $100 million principal at the six-month LIBOR rate prevailing six months prior to September 5, 2001—that is, on March 5, 2001. Suppose that six-month LIBOR rate on March 5, 2001, is 4.2%. Intel pays Microsoft $0.5 \times 0.042 \times \$100 = \$2.1$ million.[1] Note that there is no uncertainty about this first exchange of payments because it is determined by the LIBOR rate at the time the contract is entered into.

The second exchange of payments would take place on March 5, 2002, one year after the initiation of the agreement. Microsoft would pay $2.5 million to Intel. Intel would pay interest on the $100 million principal to Microsoft at the six-month LIBOR rate prevailing six months prior to March 5, 2002—that is, on September 5, 2001. Suppose that the six-month LIBOR rate on September 5, 2001, is 4.8%. Intel pays $0.5 \times 0.048 \times \$100 = \$2.4$ million to Microsoft.

In total, there are six exchanges of payment on the swap. The fixed payments are always $2.5 million. The floating-rate payments on a payment date are calculated using the six-month LIBOR rate prevailing six months before the payment date. An interest rate swap is generally structured so that one side remits the difference between the two payments to the other side. In our example, Microsoft would pay Intel $0.4 million (= $2.5 million − $2.1 million) on September 5, 2001, and $0.1 million (= $2.5 million − $2.4 million) on March 5, 2002.

Table 6.1 provides a complete example of the payments made under the swap for one particular set of six-month LIBOR rates. The table shows the swap cash flows from the

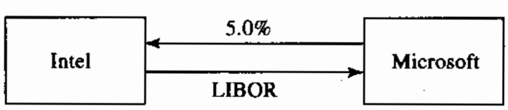

Figure 6.1 Interest rate swap between Microsoft and Intel

[1] The calculations here are slightly inaccurate because they ignore day count conventions. This point is discussed in more detail later in the chapter.

Table 6.1 Cash flows (millions of dollars) to Microsoft in a $100 million three-year interest rate swap when a fixed rate of 5% is paid and LIBOR is received

Date	6-month LIBOR rate (%)	Floating cash flow received	Fixed cash flow paid	Net cash flow
Mar. 5, 2001	4.20			
Sept. 5, 2001	4.80	+2.10	−2.50	−0.40
Mar. 5, 2002	5.30	+2.40	−2.50	−0.10
Sept. 5, 2002	5.50	+2.65	−2.50	+0.15
Mar. 5, 2003	5.60	+2.75	−2.50	+0.25
Sept. 5, 2003	5.90	+2.80	−2.50	+0.30
Mar. 5, 2004	6.40	+2.95	−2.50	+0.45

perspective of Microsoft. Note that the $100 million principal is used only for the calculation of interest payments. The principal itself is not exchanged. This is why it is termed the *notional principal.*

If the principal were exchanged at the end of the life of the swap, the nature of the deal would not be changed in any way. The principal is the same for both the fixed and floating payments. Exchanging $100 million for $100 million at the end of the life of the swap is a transaction that would have no financial value to either Microsoft or Intel. Table 6.2 shows the cash flows in Table 6.1 with a final exchange of principal added in. This provides an interesting way of viewing the swap. The cash flows in the third column of this table are the cash flows from a long position in a floating-rate bond. The cash flows in the fourth column of the table are the cash flows from a short position in a fixed-rate bond. The table shows that the swap can be regarded as the exchange of a fixed-rate bond for a floating-rate bond. Microsoft, whose position is described by Table 6.2, is long a floating-rate bond and short a fixed-rate bond. Intel is long a fixed-rate bond and short a floating-rate bond.

This characterization of the cash flows in the swap helps to explain why the floating rate in the swap is set six months before it is paid. On a floating-rate note, interest is generally set at the beginning of the period to which it will apply and is paid at the end of the period. The calculation of the floating-rate payments in a "plain vanilla" interest rate swap such as the one in Table 6.2 reflects this.

Table 6.2 Cash flows (millions of dollars) from Table 6.1 when there is a final exchange of principal

Date	6-month LIBOR rate (%)	Floating cash flow received	Fixed cash flow paid	Net cash flow
Mar. 5, 2001	4.20			
Sept. 5, 2001	4.80	+2.10	−2.50	−0.40
Mar. 5, 2002	5.30	+2.40	−2.50	−0.10
Sept. 5, 2002	5.50	+2.65	−2.50	+0.15
Mar. 5, 2003	5.60	+2.75	−2.50	+0.25
Sept. 5, 2003	5.90	+2.80	−2.50	+0.30
Mar. 5, 2004	6.40	+102.95	−102.50	+0.45

Using the Swap to Transform a Liability

For Microsoft, the swap could be used to transform a floating-rate loan into a fixed-rate loan. Suppose that Microsoft has arranged to borrow $100 million at LIBOR plus 10 basis points. (One basis point is one-hundredth of 1%, so the rate is LIBOR plus 0.1%.) After Microsoft has entered into the swap, it has three sets of cash flows:

1. It pays LIBOR plus 0.1% to its outside lenders.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate payment of 5.1%. Thus, for Microsoft the swap could have the effect of transforming borrowings at a floating rate of LIBOR plus 10 basis points into borrowings at a fixed rate of 5.1%.

For Intel the swap could have the effect of transforming a fixed-rate loan into a floating-rate loan. Suppose that Intel has a three-year $100 million loan outstanding on which it pays 5.2%. After it has entered into the swap, it has three sets of cash flows:

1. It pays 5.2% to its outside lenders.
2. It pays LIBOR under the terms of the swap.
3. It receives 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate payment of LIBOR plus 0.2% (or LIBOR plus 20 basis points). Thus, for Intel the swap could have the effect of transforming borrowings at a fixed rate of 5.2% into borrowings at a floating rate of LIBOR plus 20 basis points. These potential uses of the swap by Intel and Microsoft are illustrated in Figure 6.2.

Using the Swap to Transform an Asset

Swaps can also be used to transform the nature of an asset. Consider Microsoft in our example. The swap could have the effect of transforming an asset earning a fixed rate of interest into an asset earning a floating rate of interest. Suppose that Microsoft owns $100 million in bonds that will provide interest at 4.7% per annum over the next three years. After Microsoft has entered into the swap, it has three sets of cash flows:

1. It receives 4.7% on the bonds.
2. It receives LIBOR under the terms of the swap.
3. It pays 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate inflow of LIBOR minus 30 basis points. Thus, one possible use of the swap for Microsoft is to transform an asset earning 4.7% into an asset earning LIBOR minus 30 basis points.

Consider next Intel. The swap could have the effect of transforming an asset earning a floating rate of interest into an asset earning a fixed rate of interest. Suppose that Intel

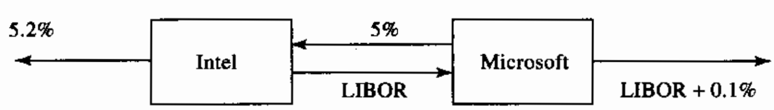

Figure 6.2 Microsoft and Intel use the swap to transform a liability

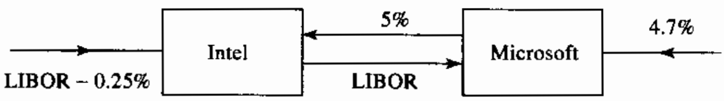

Figure 6.3 Microsoft and Intel use the swap to transform an asset

has an investment of $100 million that yields LIBOR minus 25 basis points. After it has entered into the swap, it has three sets of cash flows:

1. It receives LIBOR minus 25 basis points on its investment.
2. It pays LIBOR under the terms of the swap.
3. It receives 5% under the terms of the swap.

These three sets of cash flows net out to an interest rate inflow of 4.75%. Thus, one possible use of the swap for Intel is to transform an asset earning LIBOR minus 25 basis points into an asset earning 4.75%. These potential uses of the swap by Intel and Microsoft are illustrated in Figure 6.3.

Role of Financial Intermediary

Usually two nonfinancial companies such as Intel and Microsoft do not get in touch directly to arrange a swap in the way indicated in Figures 6.2 and 6.3. They each deal with a financial intermediary such as a bank or other financial institution. "Plain vanilla" fixed-for-floating swaps on U.S. interest rates are usually structured so that the financial institution earns about 3 or 4 basis points (0.03 to 0.04%) on a pair of offsetting transactions.

Figure 6.4 shows what the role of the financial institution might be in the situation in Figure 6.2. The financial institution enters into two offsetting swap transactions with Intel and Microsoft. Assuming that neither defaults, the financial institution is certain to make a profit of 0.03% (3 basis points) per year multiplied by the notional principal of $100 million. (This amounts to $30,000 per year for the three-year period.) Microsoft ends up borrowing at 5.115% (instead of 5.1%, as in Figure 6.2). Intel ends up borrowing at LIBOR plus 21.5 basis points (instead of at LIBOR plus 20 basis points, as in Figure 6.2).

Figure 6.5 illustrates the role of the financial institution in the situation in Figure 6.3. Again, the financial institution is certain to make a profit of three basis points if neither company defaults on the swap. Microsoft ends up earning LIBOR minus 31.5 basis points (instead of LIBOR minus 30 basis points, as in Figure 6.3). Intel ends up earning 4.735% (instead of 4.75%, as in Figure 6.3).

Note that in each case the financial institution has two separate contracts: one with Intel and the other with Microsoft. In most instances, Intel will not even know that the financial institution has entered into an offsetting swap with Microsoft, and vice versa. If one of the companies defaults, the financial institution still has to honor its agreement with the other company. The three-basis-point spread earned by the financial institution is partly to compensate it for the default risk it is bearing.

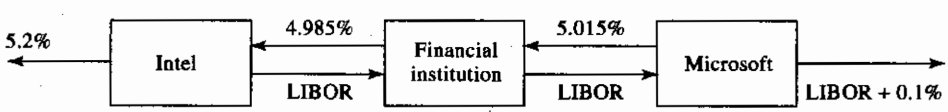

Figure 6.4 Interest rate swap from Figure 6.2 when financial institution is used

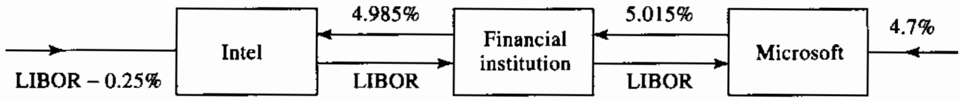

Figure 6.5 Interest rate swap from Figure 6.3 when financial institution is used

Warehousing

In practice, it is unlikely that two companies will contact a financial institution at the same time and want to take opposite positions in exactly the same swap. For this reason, many large financial institutions are prepared to enter into a swap without having an offsetting swap with another counterparty. This is sometimes referred to as *warehousing* interest rate swaps. The financial institutions must carefully quantify and hedge the risks it is taking. Bonds, forward rate agreements, and interest rate futures are examples of the instruments that can be used for hedging. The way these financial institutions act as market makers and provide quotes in the swap market is discussed later in this chapter.

Day Count Conventions

The day count conventions discussed in Section 5.8 affect payments on a swap, and some of the numbers calculated in the example we have given do not exactly reflect these day count conventions. Consider, for example, the six-month LIBOR payments in Table 6.1. Because it is a money market rate, six-month LIBOR is generally quoted on an actual/360 basis. The first floating payment in Table 6.1, based on the LIBOR rate of 4.2%, is shown as $2.10 million. Because there are 184 days between March 5, 2001, and September 5, 2001, it should be

$$100 \times 0.042 \times \frac{184}{360} = \$2.1467 \text{ million}$$

In general, a LIBOR-based floating-rate cash flow on a swap payment date is calculated as $LRn/360$, where L is the principal, R is the relevant LIBOR rate, and n is the number of days since the last payment date.

The fixed rate that is paid in a swap transaction is similarly quoted with a particular day count basis being specified. As a result, the fixed payments may not be exactly equal on each payment date. The fixed rate is usually quoted as actual/365 or 30/360. It is not therefore directly comparable with LIBOR because it applies to a full year. To make the rates comparable, either the six-month LIBOR rate must be multiplied by 365/360 or the fixed rate must be multiplied by 360/365.

For ease of exposition we will ignore day count issues in the examples in the rest of this chapter.

Confirmations

A *confirmation* is the legal agreement underlying a swap and is signed by representatives of the two parties. Table 6.3 could be an extract from the confirmation between Microsoft and Intel. The drafting of confirmations has been facilitated by the work of the International Swaps and Derivatives Association (ISDA) in New York. This organization has produced a number of Master Agreements that consist of clauses defining in some detail the terminology used in swap agreements, what happens in the

Table 6.3 Extract from confirmation for a plain vanilla swap between Microsoft and Intel

Trade date	27-February-2001
Effective date	5-March-2001
Business day convention (all dates)	Following business day
Holiday calendar	U.S.
Termination date	5-March-2006
Fixed amounts	
Fixed rate payer	Microsoft
Fixed rate notional principal	USD 100 million
Fixed rate	5% per annum
Fixed rate day count convention	Actual/365
Fixed rate payment dates	Each 5-March and 5-September commencing 5-September, 2001, up to and including 5-March, 2006
Floating amounts	
Floating rate payer	Intel
Floating rate notional principal	USD 100 million
Floating rate	USD 6-month LIBOR
Floating rate day count convention	Actual/360
Floating rate payment dates	Each 5-March and 5-September commencing 5-September, 2001, up to and including 5-March, 2006

event of default by either side, and so on. Almost certainly, the full confirmation for the swap in Table 6.3 would state that the provisions of an ISDA Master Agreement apply to the contract.

Table 6.3 specifies that the following business day convention is to be used and that the U.S. calendar determines which days are business days and which days are holidays. This means that, if a payment date falls on a weekend or a U.S. holiday, the payment is made on the next business day.[2] In the example in Table 6.3, September 5, 2004, is a Sunday. The payment is, therefore, made on Monday September 6, 2004.

6.2 THE COMPARATIVE-ADVANTAGE ARGUMENT

An explanation commonly put forward to explain the popularity of swaps concerns comparative advantages. Consider the use of an interest rate swap to transform a liability. Some companies, it is argued, have a comparative advantage when borrowing in fixed-rate markets, whereas other companies have a comparative advantage in floating-rate markets. To obtain a new loan, it makes sense for a company to go to the market where it has a comparative advantage. As a result, the company may borrow fixed when it wants floating, or borrow floating when it wants fixed. The swap is used to transform a fixed-rate loan into a floating-rate loan, and vice versa.

[2] Another business day convention that is somctimes specified is the *modified following* business day convention, which is the same as the following business day convention except that when the next business day falls in a different month from the specified day, the payment is made on the immediately preceding business day. *Preceding* and *modified preceding* business day conventions are defined analogously.

Table 6.4 Borrowing rates that provide a basis for the comparative-advantage argument

	Fixed	Floating
AAACorp	10.0%	6-month LIBOR + 0.3%
BBBCorp	11.2%	6-month LIBOR + 1.0%

Illustration

Suppose that two companies, AAACorp and BBBCorp, both wish to borrow $10 million for five years and have been offered the rates shown in Table 6.4. AAACorp has a AAA credit rating; BBBCorp has a BBB credit rating. We assume that BBBCorp wants to borrow at a fixed rate of interest, whereas AAACorp wants to borrow at a floating rate of interest linked to six-month LIBOR. BBBCorp, because of its worse credit rating than AAACorp, pays a higher rate of interest than AAACorp in both fixed and floating markets.

A key feature of the rates offered to AAACorp and BBBCorp is that the difference between the two fixed rates is greater than the difference between the two floating rates. BBBCorp pays 1.2% more than AAACorp in fixed-rate markets and only 0.7% more than AAACorp in floating-rate markets. BBBCorp appears to have a comparative advantage in floating-rate markets, whereas AAACorp appears to have a comparative advantage in fixed-rate markets.[3] It is this apparent anomaly that can lead to a swap being negotiated. AAACorp borrows fixed-rate funds at 10% per annum. BBBCorp borrows floating-rate funds at LIBOR plus 1% per annum. They then enter into a swap agreement to ensure that AAACorp ends up with floating-rate funds and BBBCorp ends up with fixed-rate funds.

To understand how the swap might work, we first assume that AAACorp and BBBCorp get in touch with each other directly. The sort of swap they might negotiate is shown in Figure 6.6. This is very similar to our example in Figure 6.2. AAACorp agrees to pay BBBCorp interest at six-month LIBOR on $10 million. In return, BBBCorp agrees to pay AAACorp interest at a fixed rate of 9.95% per annum on $10 million.

AAACorp has three sets of interest rate cash flows:

1. It pays 10% per annum to outside lenders.
2. It receives 9.95% per annum from BBBCorp.
3. It pays LIBOR to BBBCorp.

The net effect of the three cash flows is that AAACorp pays LIBOR plus 0.05% per

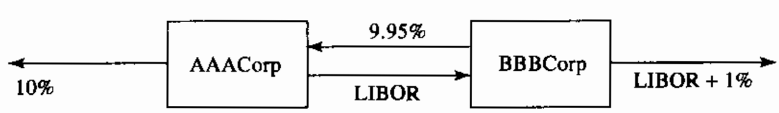

Figure 6.6 Swap agreement between AAACorp and BBBCorp when rates in Table 6.4 apply

[3] Note that BBBCorp's comparative advantage in floating-rate markets does not imply that BBBCorp pays less than AAACorp in this market. It means that the extra amount that BBBCorp pays over the amount paid by AAACorp is less in this market. One of my students summarized the situation as follows: "AAACorp pays more less in fixed-rate markets; BBBCorp pays less more in floating-rate markets."

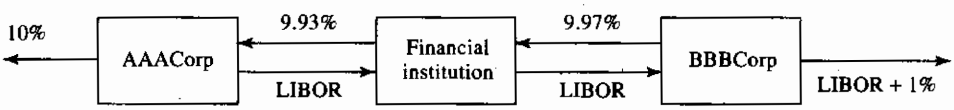

Figure 6.7 Swap agreement between AAACorp and BBBCorp when rates in Table 6.4 apply and a financial intermediary is involved

annum. This is 0.25% per annum less than it would pay if it went directly to floating-rate markets. BBBCorp also has three sets of interest rate cash flows:

1. It pays LIBOR + 1% per annum to outside lenders.
2. It receives LIBOR from AAACorp.
3. It pays 9.95% per annum to AAACorp.

The net effect of the three cash flows is that BBBCorp pays 10.95% per annum. This is 0.25% per annum less than it would pay if it went directly to fixed-rate markets.

The swap arrangement appears to improve the position of both AAACorp and BBBCorp by 0.25% per annum. The total gain is therefore 0.5% per annum. It can be shown that the total apparent gain from this type of interest rate swap arrangement is always $a - b$, where a is the difference between the interest rates facing the two companies in fixed-rate markets, and b is the difference between the interest rates facing the two companies in floating-rate markets. In this case, $a = 1.2\%$ and $b = 0.70\%$.

If AAACorp and BBBCorp did not deal directly with each other and used a financial institution, an arrangement such as that shown in Figure 6.7 might result. (This is very similar to the example in Figure 6.4.) In this case, AAACorp ends up borrowing at LIBOR + 0.07%, BBBCorp ends up borrowing at 10.97%, and the financial institution earns a spread of four basis points per year. The gain to AAACorp is 0.23%; the gain to BBBCorp is 0.23%; and the gain to the financial institution is 0.04%. The total gain to all three parties is 0.50% as before. Table 6.5 summarizes this example.

Criticism of the Comparative-Advantage Argument

The comparative-advantage argument we have just outlined for explaining the attractiveness of interest rate swaps is open to question. Why in Table 6.4 should the spreads between the rates offered to AAACorp and BBBCorp be different in fixed and floating markets? Now that the swap market has been in existence for some time, we might reasonably expect these types of differences to have been arbitraged away.

The reason that spread differentials appear to exist is due to the nature of the contracts available to companies in fixed and floating markets. The 10.0% and 11.2% rates available to AAACorp and BBBCorp in fixed-rate markets are five-year rates (for example, the rates at which the companies can issue five-year fixed-rate bonds). The LIBOR + 0.3% and LIBOR + 1.0% rates available to AAACorp and BBBCorp in floating-rate markets are six-month rates. In the floating-rate market, the lender usually has the opportunity to review the floating rates every six months. If the creditworthiness of AAACorp or BBBCorp has declined, the lender has the option of increasing the spread over LIBOR that is charged. In extreme circumstances, the lender can refuse to roll over the loan at all. The providers of fixed-rate financing do not have the option to change the terms of the loan in this way.[4]

[4] If the floating rate loans are structured so that the spread over LIBOR is guaranteed in advance regardless of changes in credit rating, there is in practice little or no comparative advantage.

Table 6.5 An interest rate swap arrangement based on apparent comparative advantages

From the Trader's Desk

AAACorp and BBBCorp both want to borrow $10 million for five years. AAACorp wants to arrange a floating-rate loan in which the rate of interest is linked to six-month LIBOR. BBBCorp wants to arrange a fixed-rate loan. They have been offered the following terms.

	Fixed	Floating
AAACorp	10.0%	6-month LIBOR + 0.3%
BBBCorp	11.2%	6-month LIBOR + 1.0%

The Strategy

1. AAACorp borrows fixed-rate funds at 10% per annum.
2. BBBCorp borrows floating-rate funds at LIBOR + 1% per annum.
3. They then enter into a swap agreement.

The Swap with No Intermediary

The arrangement is shown in Figure 6.6. AAACorp agrees to pay BBBCorp the six-month LIBOR rate of interest on $10 million. In return, BBBCorp agrees to pay AAACorp 9.95% per annum on $10 million. The net result is that AAACorp ends up borrowing at LIBOR + 0.05%, whereas BBBCorp ends up borrowing at 10.95%. The swap appears to make both sides 0.25% per annum better off.

The Swap with Intermediary

The arrangement is shown in Figure 6.7. Each of AAACorp and BBBCorp enters into a swap agreement with a financial intermediary. AAACorp ends up borrowing at LIBOR + 0.07% per annum, BBBCorp ends up borrowing at 10.97% per annum, and the intermediary achieves a spread of 0.04% per annum. The swap appears to make each of AAACorp and BBBCorp 0.23% per annum better off.

The spreads between the rates offered to AAACorp and BBBCorp are a reflection of the extent to which BBBCorp is more likely to default than AAACorp. During the next six months, there is very little chance that either AAACorp or BBBCorp will default. As we look further ahead, default statistics show that the probability of a default by a company with a relatively low credit rating (such as BBBCorp) increases faster than the probability of a default by a company with a relatively high credit rating (such as AAACorp). This is why the spread between the five-year rates is greater than the spread between the six-month rates.

After negotiating a floating-rate loan at LIBOR + 1.0% and entering into the swap shown in Figure 6.7, BBBCorp appears to obtain a fixed-rate loan at 10.97%. The arguments just presented show that this is not really the case. In practice, the rate paid is 10.97% only if BBBCorp can continue to borrow floating-rate funds at a spread of 1.0% over LIBOR. If, for example, the credit rating of BBBCorp declines so that the floating-rate loan is rolled over at LIBOR + 2.0%, the rate paid by BBBCorp increases to 11.97%. Because BBBCorp's spread over six-month LIBOR is more likely to rise than to fall, BBBCorp's expected average borrowing rate when it enters into the swap is greater than 10.97%.

The swap in Figure 6.7 locks in LIBOR + 0.07% for AAACorp for the whole of the next five years, not just for the next six months. This appears to be a good deal for AAACorp. The downside is that it is bearing the risk of a default by the financial institution. If it borrowed floating-rate funds in the usual way it would not be bearing this risk.

6.3 SWAP QUOTES AND LIBOR ZERO RATES

We now return to the interest rate swap in Figure 6.1. We showed in Table 6.2 that it can be characterized as the difference between two bonds. Although the principal is not exchanged, we can assume without changing the value of the swap that, at the end of the agreement, Intel pays Microsoft the notional principal of $100 million and Microsoft pays Intel the same notional principal. The swap is then the same as an arrangement in which:

1. Microsoft has lent Intel $100 million at the six-month LIBOR rate
2. Intel has lent Microsoft $100 million at a fixed rate of 5% per annum

To put it another way, Microsoft has purchased a $100 million floating-rate (LIBOR) bond from Intel and has sold a $100 million fixed-rate (5% per annum) bond to Intel. The value of the swap to Microsoft is therefore the difference between the values of two bonds. Define

B_{fix}: Value of fixed-rate bond underlying the swap

B_{fl}: Value of floating-rate bond underlying the swap

The value of the swap to a company receiving floating and paying fixed (Microsoft in our example) is

$$V_{\text{swap}} = B_{\text{fl}} - B_{\text{fix}} \tag{6.1}$$

Swap Rates

Many large financial institutions are market makers in the swap market. This means that they are prepared to quote, for a number of different maturities and a number of different currencies, a bid and an offer for the fixed rate they will exchange for floating. The bid is the fixed rate in a contract where the market maker will pay fixed and receive floating; the offer is the fixed rate in a swap where the market maker will receive fixed and pay floating. Table 6.6 shows typical quotes for plain vanilla U.S. dollar swaps. As mentioned earlier, the bid–offer spread is three to four basis points. The average of the

Table 6.6 Bid and offer fixed rates in the swap market and swap rates (percent per annum); payments exchanged semiannually

Maturity (years)	Bid	Offer	Swap rate
2	6.03	6.06	6.045
3	6.21	6.24	6.225
4	6.35	6.39	6.370
5	6.47	6.51	6.490
7	6.65	6.68	6.665
10	6.83	6.87	6.850

bid and offer fixed rates is known as the *swap rate*. This is shown in the final column of Table 6.6.

Consider a new swap where the fixed rate equals the swap rate. We can reasonably assume that the value of this swap is zero. (Why else would a market maker choose bid–offer quotes centered around the swap rate?) From equation (6.1) it follows that

$$B_{\text{fix}} = B_{\text{fl}} \tag{6.2}$$

As mentioned in Section 5.1, banks and other financial institutions usually discount cash flows in the over-the-counter market at LIBOR rates of interest. The floating-rate bond underlying the swap pays LIBOR. As a result, the value of this bond, B_{fl}, equals the swap principal. It follows from equation (6.2) that the value of the fixed-rate bond, B_{fix} also equals the swap principal. A swap rate is therefore a LIBOR par yield. It is the coupon rate on the LIBOR bond that causes it to be worth par.

Determining the LIBOR Zero Curve

In Section 5.11 we showed how Eurodollar futures can be used to determine LIBOR zero rates. Swap rates also play an important role in determining LIBOR zero rates. As we have just seen, they define a series of LIBOR par yield bonds. The latter can be used to bootstrap a LIBOR zero curve in the same way that Treasury bonds are used to bootstrap the Treasury zero curve. (See Section 5.4.)

Example

Assume that the LIBOR zero curve has already been calculated out to 1.5 years (using spot LIBOR rates and Eurodollar futures) and that we wish to use the swap rates in Table 6.6 to extend the curve. The 6-month, 1-year, and 1.5-year zero rates are, respectively, 5.5%, 5.75%, and 5.9% per annum with continuous compounding. Because the swaps in Table 6.6 involve semiannual cash flows, we first interpolate between the swap rates to obtain swap rates at intervals of 0.5 years. The 2.5-year swap rate is 6.135%, the 3.5-year swap rate is 6.2975%, and so on. Next we use the bootstrap method described in Section 5.4. Because the 2-year swap rate is the two-year par yield, a 2-year bond paying a semiannual coupon of 6.045% per annum must sell for par so that

$$3.0225e^{-0.055\times 0.5} + 3.0225e^{-0.0575\times 1.0} + 3.0225e^{-0.059\times 1.5} + 103.0225e^{-2R} = 100$$

where R is the 2-year zero rate. This can be solved to give $R = 5.9636\%$. Similarly, a 2.5-year bond paying a semiannual coupon of 6.135% must sell for par so that

$$3.0675e^{-0.055\times 0.5} + 3.0675e^{-0.0575\times 1.0} + 3.0675e^{-0.059\times 1.5} + 3.0675e^{-0.059636\times 2} + 103.0675e^{-2.5R} = 100$$

where R is the 2.5-year zero rate. This can be solved to get $R = 6.0549\%$. Continuing in this way the complete term structure is obtained. The 3-, 4-, 5-, 7-, and 10-year zero rates are 6.1475%, 6.2986%, 6.4265%, 6.6189%, and 6.8355%, respectively.

In the United States, spot LIBOR rates are typically used to define the LIBOR zero curve for maturities up to one year. Eurodollar futures are then used for maturities between one and two years—and sometimes for maturities up to five years. Swap rates are used to calculate the zero curve for longer maturities. A similar procedure is followed to determine LIBOR zero rates in other countries. For example, Swiss franc LIBOR zero rates are determined from spot Swiss franc LIBOR rates, three-month Euroswiss futures, and Swiss franc swap rates.

6.4 VALUATION OF INTEREST RATE SWAPS

An interest rate swap is worth zero, or close to zero, when it is first initiated. After it has been in existence for some time, its value may become positive or negative. To calculate the value we can regard the swap either as a long position in one bond combined with a short position in another bond or as a portfolio of forward rate agreements. In either case we use LIBOR zero rates for discounting.

Valuation in Terms of Bond Prices

We saw in equation (6.1) that for a swap where floating is received and fixed is paid

$$V_{\text{swap}} = B_{\text{fl}} - B_{\text{fix}}$$

In Section 6.3 we used this equation to show that B_{fix} equals the swap's notional principal at the initiation of the swap. We now use it to value the swap some time after its initiation. To see how this equation is used, we define

- t_i: Time until ith payments are exchanged ($1 \leqslant i \leqslant n$)
- L: Notional principal in swap agreement
- r_i: LIBOR zero rate corresponding to maturity t_i
- k: Fixed payment made on each payment date

The fixed-rate bond, B_{fix}, can be valued as described in Section 5.3. The cash flows from the bond are k at time t_i ($1 \leqslant i \leqslant n$) and L at time t_n so that

$$B_{\text{fix}} = \sum_{i=1}^{n} k e^{-r_i t_i} + L e^{-r_n t_n}$$

Consider next the floating-rate bond. Immediately after a payment date this is identical to a newly issued floating-rate bond. It follows that $B_{\text{fl}} = L$ immediately after a payment date. Between payment dates, we can use the fact that B_{fl} will equal L immediately after the next payment date and argue as follows. Immediately before the next payment date $B_{\text{fl}} = L + k^*$, where k^* is the floating-rate payment (already known) that will be made on the next payment date. In our notation, the time until the next payment date is t_1. The value of the swap today is its value just before the next payment date discounted at rate r_1 for time t_1:

$$B_{\text{fl}} = (L + k^*)e^{-r_1 t_1}$$

In the situation where the company is receiving fixed and paying floating, B_{fix} and B_{fl} are calculated in the same way, and equation (6.1) becomes

$$V_{\text{swap}} = B_{\text{fix}} - B_{\text{fl}} \tag{6.3}$$

Example

Suppose that, under the terms of a swap, a financial institution has agreed to pay six-month LIBOR and receive 8% per annum (with semiannual compounding) on a notional principal of $100 million. The swap has a remaining life of 1.25 years. The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month LIBOR rate at the last payment date was 10.2% (with semiannual compounding). In this case, $k = \$4$ million and $k^* = \$5.1$ million, so that

$$\begin{aligned} B_{\text{fix}} &= 4e^{-0.1\times 3/12} + 4e^{-0.105\times 9/12} + 104e^{-0.11\times 15/12} \\ &= \$98.24 \text{ million} \\ B_{\text{fl}} &= 5.1e^{-0.1\times 3/12} + 100e^{-0.1\times 3/12} \\ &= \$102.51 \text{ million} \end{aligned}$$

Hence, the value of the swap is

$$98.24 - 102.51 = -\$4.27 \text{ million}$$

If the bank had been in the opposite position of paying fixed and receiving floating, the value of the swap would be +$4.27 million. Note that a more precise calculation would take account of the actual/360 day count convention in calculating k^*.

Valuation in Terms of Forward Rate Agreements

We introduced forward rate agreements (FRAs) in Chapter 5. They are agreements that a certain predetermined interest rate will apply to a certain principal for a certain period of time in the future. In Section 5.6 we showed that an FRA can be characterized as an agreement where interest at the predetermined rate is exchanged for interest at the market rate of interest for the period in question. This shows that an interest rate swap is nothing more than a portfolio of forward rate agreements.

Consider again the swap agreement between Intel and Microsoft in Figure 6.1. As illustrated in Table 6.1 this commits Microsoft to six cash flow exchanges. The first exchange is known at the time the swap is negotiated. The other five exchanges can be regarded as FRAs. The exchange on March 5, 2002, is an FRA where interest at 5% is exchanged for interest at the six-month rate observed in the market on September 5, 2001; the exchange on September 5, 2002, is an FRA where interest at 5% is exchanged for interest at the six-month rate observed in the market on March 5, 2002; and so on.

As shown in Section 5.6, an FRA can be valued by assuming that forward interest rates are realized. Because it is a portfolio of forward rate agreements, a plain vanilla interest rate swap can also be valued by making the assumption that forward interest rates are realized. The procedure is as follows:

1. Calculate forward rates for each of the LIBOR rates that will determine swap cash flows.
2. Calculate swap cash flows on the assumption that the LIBOR rates will equal the forward rates.
3. Set the swap value equal to the present value of these cash flows.

Example

Consider again the situation in the previous example. The cash flows that will be exchanged in 3 months have already been determined. A rate of 8% will be exchanged for 10.2%. The value of the exchange to the financial institution is

$$0.5 \times 100 \times (0.08 - 0.102)e^{-0.1\times 3/12} = -1.07$$

To calculate the value of the exchange in 9 months, we must first calculate the forward rate corresponding to the period between 3 and 9 months. From equation (5.1), this is

$$\frac{0.105 \times 0.75 - 0.10 \times 0.25}{0.5} = 0.1075$$

or 10.75% with continuous compounding. From equation (3.4), this value becomes 11.044% with semiannual compounding. The value of the FRA corresponding to the exchange in 9 months is therefore

$$0.5 \times 100 \times (0.08 - 0.11044)e^{-0.105\times 9/12} = -1.41$$

To calculate the value of the exchange in 15 months, we must first calculate the forward rate corresponding to the period between 9 and 15 months. From equation (5.1), this is

$$\frac{0.11 \times 1.25 - 0.105 \times 0.75}{0.5} = 0.1175$$

or 11.75% with continuous compounding. From equation (3.4), this value becomes 12.102% with semiannual compounding. The value of the FRA corresponding to the exchange in 15 months is therefore

$$0.5 \times 100 \times (0.08 - 0.12102)e^{-0.11\times 15/12} = -1.79$$

The total value of the swap is

$$-1.07 - 1.41 - 1.79 = -4.27$$

or −$4.27 million. This is in agreement with our earlier calculation based on bond prices.

As already mentioned, the fixed rate in an interest rate swap is chosen so that the swap is worth zero initially. This means that the sum of the value of the FRAs underlying the swap is zero. It does not mean that the value of each individual FRA is zero. In general, some FRAs will have positive values whereas others have negative values.

Consider the FRAs underlying the swap between the financial institution and Microsoft in Figure 6.4.

Value of FRA to financial institution < 0 when forward interest rate > 5.015%

Value of FRA to financial institution = 0 when forward interest rate = 5.015%

Value of FRA to financial institution > 0 when forward interest rate < 5.015%

Suppose that the term structure is upward sloping at the time the swap is negotiated. This means that the forward interest rates increase as the maturity of the FRA increases. Because the sum of the values of the FRAs is zero, the forward interest rate must be less than 5.015% for the early payment dates and greater than 5.015% for the

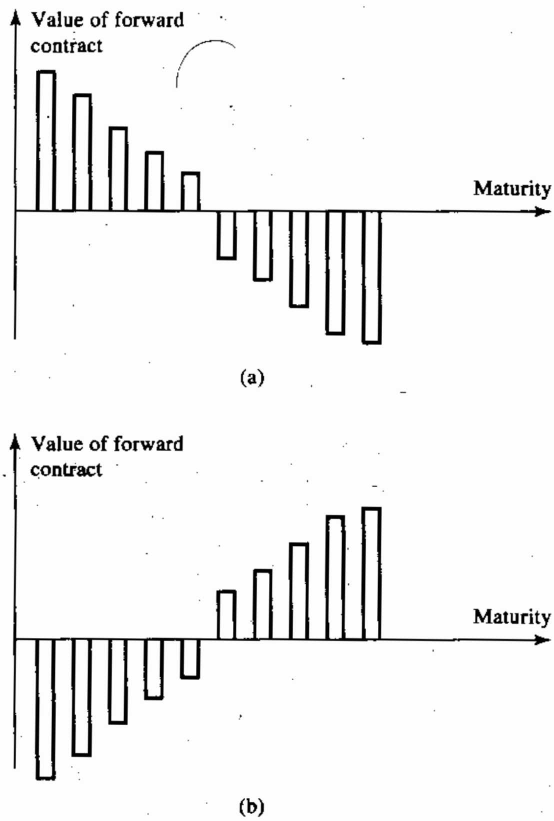

Figure 6.8 Value of forward contracts underlying a swap as a function of maturity. In (a) the yield curve is upward sloping and we receive fixed, or the yield curve is downward sloping and we receive floating; in (b) the yield curve is upward sloping and we receive floating, or the yield curve is downward sloping and we receive fixed

later payment dates. The value to the financial institution of the FRAs corresponding to early payment dates is therefore positive, whereas the value of the FRAs corresponding to later payment dates is negative. If the term structure is downward sloping at the time the swap is negotiated, the reverse is true. The impact of the shape of the term structure on the values of the forward contracts underlying a swap is summarized in Figure 6.8.

6.5 CURRENCY SWAPS

Another popular type of swap is known as a *currency swap*. In its simplest form, this involves exchanging principal and interest payments in one currency for principal and interest payments in another currency.

A currency swap agreement requires the principal to be specified in each of the two currencies. The principal amounts are usually exchanged at the beginning and at the end of the life of the swap. Usually the principal amounts are chosen to be approximately equivalent using the exchange rate at the swap's initiation.

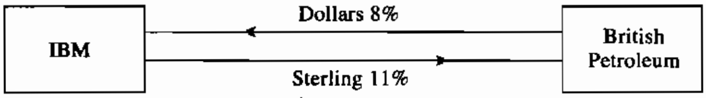

Figure 6.9 A currency swap

Illustration

Consider a hypothetical five-year currency swap agreement between IBM and British Petroleum entered into on February 1, 2001. We suppose that IBM pays a fixed rate of interest of 11% in sterling and receives a fixed rate of interest of 8% in dollars from British Petroleum. Interest rate payments are made once a year and the principal amounts are $15 million and £10 million. This is termed a *fixed-for-fixed* currency swap because the interest rate in both currencies is fixed. The swap is shown in Figure 6.9. Initially, the principal amounts flow in the opposite direction to the arrows in Figure 6.9. The interest payments during the life of the swap and the final principal payment flow in the same direction as the arrows. Thus, at the outset of the swap, IBM pays $15 million and receives £10 million. Each year during the life of the swap contract, IBM receives $1.20 million (= 8% of $15 million) and pays £1.10 million (= 11% of £10 million). At the end of the life of the swap, it pays a principal of £10 million and receives a principal of $15 million. These cash flows are shown in Table 6.7.

Use of a Currency Swap to Transform Loans and Assets

A swap such as the one just considered can be used to transform borrowings in one currency to borrowings in another currency. Suppose that IBM can issue $15 million of U.S.-dollar-denominated bonds at 8% interest. The swap has the effect of transforming this transaction into one where IBM has borrowed £10 million pounds at 11% interest. The initial exchange of principal converts the proceeds of the bond issue from U.S. dollars to sterling. The subsequent exchanges in the swap have the effect of swapping the interest and principal payments from dollars to sterling.

The swap can also be used to transform the nature of assets. Suppose that IBM can invest £10 million pounds in the U.K. to yield 11% per annum for the next five years, but feels that the U.S. dollar will strengthen against sterling and prefers a U.S.-denominated investment. The swap has the effect of transforming the U.K. investment into a $15 million investment in the U.S. yielding 8%.

Table 6.7 Cash flows to IBM in currency swap

Date	Dollar cash flow (millions)	Sterling cash flow (millions)
February 1, 2001	−15.00	+10.00
February 1, 2002	+1.20	−1.10
February 1, 2003	+1.20	−1.10
February 1, 2004	+1.20	−1.10
February 1, 2005	+1.20	−1.10
February 1, 2006	+16.20	−11.10

Table 6.8 Borrowing rates providing basis for currency swap

	USD*	AUD*
General Motors	5.0%	12.6%
Qantas Airways	7.0%	13.0%

* Quoted rates have been adjusted to reflect the differential impact of taxes.

Comparative Advantage

Currency swaps can be motivated by comparative advantage. To illustrate this, we consider another hypothetical example. Suppose the five-year fixed-rate borrowing costs to General Motors and Qantas Airways in U.S. dollars (USD) and Australian dollars (AUD) are as shown in Table 6.8. The data in the table suggest that Australian rates are higher than U.S. interest rates. Also, General Motors is more creditworthy than Qantas Airways, because it is offered a more favorable rate of interest in both currencies. From the viewpoint of a swap trader, the interesting aspect of Table 6.8 is that the spreads between the rates paid by General Motors and Qantas Airways in the two markets are not the same. Qantas Airways pays 2% more than General Motors in the U.S. dollar market and only 0.4% more in the AUD market.

This situation is analogous to that in Table 6.4. General Motors has a comparative advantage in the USD market, whereas Qantas Airways has a comparative advantage in the AUD market. In Table 6.4, where a plain vanilla interest rate swap was considered, we argued that comparative advantages were largely illusory. Here we are comparing the rates offered in two different currencies, and it is more likely that the comparative advantages are genuine. One possible source of comparative advantage is tax. General Motors's position might be such that USD borrowings lead to lower taxes on its worldwide income than AUD borrowings. Qantas Airways's position might be the reverse. (Note that we assume that the interest rates in Table 6.8 have been adjusted to reflect these types of tax advantages.)

We suppose that General Motors wants to borrow AUD and Qantas Airways wants to borrow USD. This creates a perfect situation for a currency swap. General Motors and Qantas Airways each borrow in the market where they have a comparative advantage; that is, General Motors borrows USD whereas Qantas Airways borrows AUD. They then use a currency swap to transform General Motor's loan into a AUD loan and Qantas Airway's loan into a USD loan.

As already mentioned, the difference between the dollar interest rates is 2%, whereas the difference between the AUD interest rates is 0.4%. By analogy with the interest rate swap case, we expect the total gain to all parties to be $2.0 - 0.4 = 1.6\%$ per annum.

There are many ways in which the swap can be organized. Figure 6.10 shows one way swaps might be entered into with a financial institution. General Motors borrows USD and Qantas Airways borrows AUD. The effect of the swap is to transform the USD

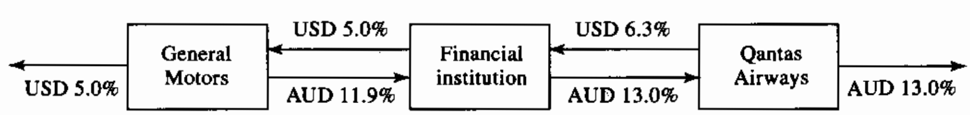

Figure 6.10 A currency swap motivated by comparative advantage

Figure 6.11 Alternative arrangement for currency swap: Qantas Airways bears some foreign exchange risk

interest rate of 5% per annum to an AUD interest rate of 11.9% per annum for General Motors. As a result, General Motors is 0.7% per annum better off than it would be if it went directly to AUD markets. Similarly, Qantas exchanges an AUD loan at 13% per annum for a USD loan at 6.3% per annum and ends up 0.7% per annum better off than it would be if it went directly to USD markets. The financial institution gains 1.3% per annum on its USD cash flows and loses 1.1% per annum on its AUD flows. If we ignore the difference between the two currencies, the financial institution makes a net gain of 0.2% per annum. As predicted, the total gain to all parties is 1.6% per annum.

Each year the financial institution makes a gain of USD 156,000 (= 1.3% of 12 million) and incurs a loss of AUD 220,000 (= 1.1% of 20 million). The financial institution can avoid any foreign exchange risk by buying AUD 220,000 per annum in the forward market for each year of the life of the swap, thus locking in a net gain in USD.

It is possible to redesign the swap so that the financial institution makes a 0.2% spread in USD. Figure 6.11 and Figure 6.12 present two alternatives. These alternatives are unlikely to be used in practice because they do not lead to General Motors and Qantas being free of foreign exchange risk.[5] In Figure 6.11 Qantas bears some foreign exchange risk because it pays 1.1% per annum in AUD and 5.2% per annum in USD. In Figure 6.12 General Motors bears some foreign exchange risk because it receives 1.1% per annum in USD and pays 13% per annum in AUD.

6.6 VALUATION OF CURRENCY SWAPS

In the absence of default risk, a currency swap can be decomposed into a position in two bonds, as is the case with an interest rate swap. Consider the position of IBM in Table 6.7 some time after the initial exchange of principal. It is short a sterling bond that pays interest at 11% per annum and long a dollar bond that pays interest at 8% per annum.

If we define V_{swap} as the value in U.S. dollars of a swap where dollars are received and

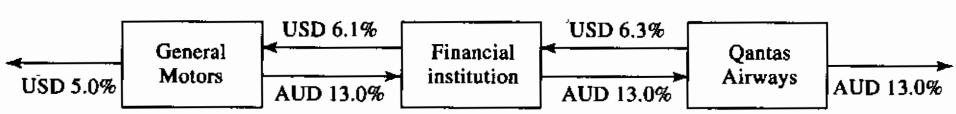

Figure 6.12 Alternative arrangement for currency swap: General Motors bears some foreign exchange risk

[5] Usually it makes sense for the financial institution to bear the foreign exchange risk, because it is in the best position to hedge the risk.

a foreign currency is paid,

$$V_{\text{swap}} = B_D - S_0 B_F$$

where B_F is the value, measured in the foreign currency, of the foreign-denominated bond underlying the swap, B_D is the value of the U.S. dollar bond underlying the swap, and S_0 is the spot exchange rate (expressed as number of units of domestic currency per unit of foreign currency). The value of a swap can therefore be determined from LIBOR rates in the two currencies, the term structure of interest rates in the domestic currency, and the spot exchange rate. Similarly, the value of a swap where the foreign currency is received and sterling is paid is

$$V_{\text{swap}} = S_0 B_F - B_D$$

Example

Suppose that the term structure of interest rates is flat in both Japan and the United States. The Japanese rate is 4% per annum and the U.S. rate is 9% per annum (both with continuous compounding). A financial institution has entered into a currency swap in which it receives 5% per annum in yen and pays 8% per annum in dollars once a year. The principals in the two currencies are \$10 million and 1,200 million yen. The swap will last for another three years, and the current exchange rate is 110 yen = \$1. In this case

$$\begin{aligned} B_D &= 0.8e^{-0.09\times 1} + 0.8e^{-0.09\times 2} + 10.8e^{-0.09\times 3} \\ &= 9.644 \text{ million dollars} \\ B_F &= 60e^{-0.04\times 1} + 60e^{-0.04\times 2} + 1,260e^{-0.04\times 3} \\ &= 1{,}230.55 \text{ million yen} \end{aligned}$$

The value of the swap in dollars is

$$\frac{1{,}230.55}{110} - 9.644 = 1.543 \text{ million}$$

If the financial institution had been paying yen and receiving dollars, the value of the swap would have been −\$1.543 million.

Decomposition into Forward Contracts

An alternative decomposition of the currency swap is into a series of forward contracts. Consider again the situation in Table 6.7. On each payment date IBM has agreed to exchange an inflow of \$1.2 million and an outflow of £1.1 million. In addition, at the final payment date, it has agreed to exchange a \$15 million inflow for a £10 million outflow. Each of these exchanges represents a forward contract. In Section 3.8 we saw that forward contracts can be valued on the assumption that the forward price of the underlying asset is realized. This provides a convenient way of valuing the forward contracts underlying a currency swap.

Example

Consider the situation in the previous example. The current spot rate is 110 yen per dollar, or 0.009091 dollar per yen. Because the difference between the dollar and yen interest rates is 5% per annum, equation (3.13) can be used to give the one-year, two-year, and three-year forward exchange rates as

$$0.009091e^{0.05\times 1} = 0.009557$$
$$0.009091e^{0.05\times 2} = 0.010047$$
$$0.009091e^{0.05\times 3} = 0.010562$$

respectively. The exchange of interest involves receiving 60 million yen and paying \$0.8 million. The risk-free interest rate in dollars is 9% per annum. From equation (3.8) the values of the forward contracts corresponding to the exchange of interest are (in millions of dollars)

$$(60 \times 0.009557 - 0.8)e^{-0.09\times 1} = -0.2071$$
$$(60 \times 0.010047 - 0.8)e^{-0.09\times 2} = -0.1647$$
$$(60 \times 0.010562 - 0.8)e^{-0.09\times 3} = -0.1269$$

The final exchange of principal involves receiving 1,200 million yen and paying \$10 million. From equation (3.8), the value of the forward contract corresponding to the exchange is (in millions of dollars)

$$(1{,}200 \times 0.010562 - 10)e^{-0.09\times 3} = 2.0416$$

The total value of the swap is 2.0416 − 0.1269 − 0.1647 − 0.2071 = \$1.543 million, which is in agreement with the result of the calculations in the previous example.

The value of a currency swap is normally zero when it is first negotiated. If the two principals are worth exactly the same using the exchange rate at the start of the swap, the value of the swap is also zero immediately after the initial exchange of principal. However, as in the case of interest rate swaps, this does not mean that each of the individual forward contracts underlying the swap has zero value. It can be shown that when interest rates in two currencies are significantly different, the payer of the low-interest rate currency is in the position where the forward contracts corresponding to the early exchanges of cash flows have positive values, and the forward contract corresponding to final exchange of principals has a negative expected value. The payer of the high-interest rate currency is likely to be in the opposite position; that is, the early exchanges of cash flows have negative values and the final exchange has a positive expected value.

For the payer of the low-interest rate currency, the swap will tend to have a negative value during most of its life. The forward contracts corresponding to the early exchanges of payments have positive values, and once these exchanges have taken place, there is a tendency for the remaining forward contracts to have, in total, a negative value. For the payer of the high-interest rate currency, the reverse is true. The value of the swap will tend to be positive during most of its life. These results are important when the credit risk in the swap is being evaluated.

6.7 CREDIT RISK

Contracts such as swaps that are private arrangements between two companies entail credit risks. Consider a financial institution that has entered into offsetting contracts with two companies (see Figure 6.4, 6.5, or 6.7). If neither party defaults, the financial institution remains fully hedged. A decline in the value of one contract will always be offset by an increase in the value of the other contract. However, there is a chance that one party will get into financial difficulties and default. The financial institution then still has to honor the contract it has with the other party.

Suppose that some time after the initiation of the contracts in Figure 6.4, the contract with Microsoft has a positive value to the financial institution, whereas the contract with Intel has a negative value. If Microsoft defaults, the financial institution is liable to lose the whole of the positive value it has in this contract. To maintain a hedged position, it would have to find a third party willing to take Microsoft's position. To induce the third party to take the position, the financial institution would have to pay the third party an amount roughly equal to the value of its contract with Microsoft prior to the default.

A financial institution has credit-risk exposure from a swap only when the value of the swap to the financial institution is positive. What happens when this value is negative and the counterparty gets into financial difficulties? In theory, the financial institution could realize a windfall gain, because a default would lead to it getting rid of a liability. In practice, it is likely that the counterparty would choose to sell the contract to a third party or rearrange its affairs in some way so that its positive value in the contract is not lost. The most realistic assumption for the financial institution is therefore as follows. If the counterparty goes bankrupt, there will be a loss if the value of the swap to the financial institution is positive, and there will be no effect on the financial institution's position if the value of the swap to the financial institution is negative. This situation is summarized in Figure 6.13.

Potential losses from defaults on a swap are much less than the potential losses from defaults on a loan with the same principal. This is because the value of the swap is usually only a small fraction of the value of the loan. Potential losses from defaults on a currency swap are greater than on an interest rate swap. The reason is that, because principal amounts in two different currencies are exchanged at the end of the life of a currency swap, a currency swap can have a greater value than an interest rate swap.

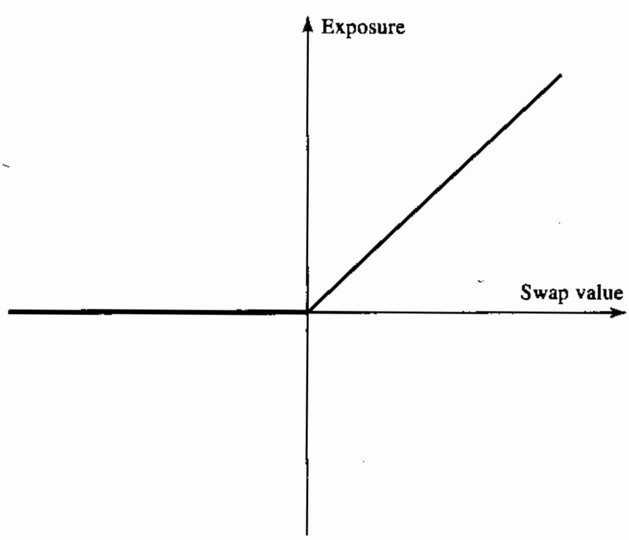

Figure 6.13 The credit exposure in a swap

Sometimes a financial institution can predict which of two offsetting contracts is more likely to have a positive value. Consider the currency swap in Figure 6.10. AUD interest rates are higher than U.S. interest rates. This means that, as time passes, the financial institution is likely to find that its swap with General Motors has a negative value whereas its swap with Qantas has a positive value. The creditworthiness of Qantas is therefore more important than the creditworthiness of General Motors.

It is important to distinguish between the credit risk and market risk to a financial institution in any contract. As discussed earlier, the credit risk arises from the possibility of a default by the counterparty when the value of the contract to the financial institution is positive. The market risk arises from the possibility that market variables such as interest rates and exchange rates will move in such a way that the value of a contract to the financial institution becomes negative. Market risks can be hedged by entering into offsetting contracts; credit risks are less easy to hedge.

6.8 SUMMARY

The two most common types of swaps are interest rate swaps and currency swaps. In an interest rate swap, one party agrees to pay the other party interest at a fixed rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time. In a currency swap, one party agrees to pay interest on a principal amount in one currency. In return, it receives interest on a principal amount in another currency.

Principal amounts are not usually exchanged in an interest rate swap. In a currency swap, principal amounts are usually exchanged at both the beginning and the end of the life of the swap. For a party paying interest in the foreign currency, the foreign principal is received, and the domestic principal is paid at the beginning of the life of the swap. At the end of the life of the swap, the foreign principal is paid and the domestic principal is received.

An interest rate swap can be used to transform a floating-rate loan into a fixed-rate loan, or vice versa. It can also be used to transform a floating-rate investment to a fixed-rate investment, or vice versa. A currency swap can be used to transform a loan in one currency into a loan in another currency. It can also be used to transform an investment denominated in one currency into an investment denominated in another currency.

There are two ways of valuing interest rate swaps and currency swaps. In the first, the swap is decomposed into a long position in one bond and a short position in another bond. In the second, it is regarded as a portfolio of forward contracts.

When a financial institution enters into a pair of offsetting swaps with different counterparties, it is exposed to credit risk. If one of the counterparties defaults when the financial institution has positive value in its swap with that counterparty, the financial institution loses money, because it still has to honor its swap agreement with the other counterparty.

The swap market is discussed further in Chapter 19.

Suggestions for Further Reading

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Turnbull, S. M. "Swaps: A Zero Sum Game," *Financial Management* 16(1) (spring 1987): 15–21.

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Quiz (Answers at End of Book)

6.1. Companies A and B have been offered the following rates per annum on a \$20 million five-year loan:

	Fixed rate	Floating rate
Company A	12.0%	LIBOR + 0.1%
Company B	13.4%	LIBOR + 0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

6.2. Company X wishes to borrow U.S. dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes:

	Yen	Dollars
Company X	5.0%	9.6%
Company Y	6.5%	10.0%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

6.3. A \$100 million interest rate swap has a remaining life of 10 months. Under the terms of the swap, six-month LIBOR is exchanged for 12% per annum (compounded semiannually). The average of the bid–offer rate being exchanged for six-month LIBOR in swaps of all maturities is currently 10% per annum with continuous compounding. The six-month LIBOR rate was 9.6% per annum two months ago. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed?

6.4. Explain what a swap rate is. What is the relationship between swap rates and par yields?

6.5. A currency swap has a remaining life of 15 months. It involves exchanging interest at 14% on £20 million for interest at 10% on \$30 million once a year. The term structure of interest rates in both the United Kingdom and the United States is currently flat, and if the swap were negotiated today the interest rates exchanged would be 8% in dollars and 11% in sterling. All interest rates are quoted with annual compounding. The current exchange rate (dollars per pound sterling) is 1.6500. What is the value of the swap to the party paying sterling? What is the value of the swap to the party paying dollars?

6.6. Explain the difference between the credit risk and the market risk in a financial contract.

6.7. Explain why a bank is subject to credit risk when it enters into two offsetting swap contracts.

Questions and Problems (Answers in Solutions Manual)

6.8. Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment:

	Fixed rate	Floating rate
Company X	8.0%	LIBOR
Company Y	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

6.9. A financial institution has entered into an interest rate swap with company X. Under the terms of the swap, it receives 10% per annum and pays six-month LIBOR on a principal of $10 million for five years. Payments are made every six months. Suppose that company X defaults on the sixth payment date (end of year 3) when the interest rate (with semiannual compounding) is 8% per annum for all maturities. What is the loss to the financial institution? Assume that six-month LIBOR was 9% per annum halfway through year 3.

6.10. A financial institution has entered into a 10-year currency swap with company Y. Under the terms of the swap, it receives interest at 3% per annum in Swiss francs and pays interest at 8% per annum in U.S. dollars. Interest payments are exchanged once a year. The principal amounts are 7 million dollars and 10 million francs. Suppose that company Y declares bankruptcy at the end of year 6, when the exchange rate is $0.80 per franc. What is the cost to the financial institution? Assume that, at the end of year 6, the interest rate is 3% per annum in Swiss francs and 8% per annum in U.S. dollars for all maturities. All interest rates are quoted with annual compounding.

6.11. Companies A and B face the following interest rates (adjusted for the differential impact of taxes):

	A	B
U.S. dollars (floating rate)	LIBOR + 0.5%	LIBOR + 1.0%
Canadian dollars (fixed rate)	5.0%	6.5%

Assume that A wants to borrow U.S. dollars at a floating rate of interest and B wants to borrow Canadian dollars at a fixed rate of interest. A financial institution is planning to arrange a swap and requires a 50-basis-point spread. If the swap is equally attractive to A and B, what rates of interest will A and B end up paying?

6.12. After it hedges its foreign exchange risk using forward contracts, is the financial institution's average spread in Figure 6.10 likely to be greater than or less than 20 basis points? Explain your answer.

6.13. "Companies with high credit risks are the ones that cannot access fixed-rate markets directly. They are the companies that are most likely to be paying fixed and receiving floating in an interest rate swap." Assume that this statement is true. Do you think it increases or decreases the risk of a financial institution's swap portfolio? Assume that companies are most likely to default when interest rates are high.

6.14. Why is the expected loss from a default on a swap less than the expected loss from the default on a loan with the same principal?

6.15. A bank finds that its assets are not matched with its liabilities. It is taking floating-rate deposits and making fixed-rate loans. How can swaps be used to offset the risk?

6.16. Explain how you would value a swap that is the exchange of a floating rate in one currency for a fixed rate in another currency.

6.17. The LIBOR zero curve is flat at 5% (continuously compounded) out to 1.5 years. Swap rates for 2- and 3-year semiannual pay swaps are 5.4% and 5.6%, respectively. Estimate the LIBOR zero rates for maturities of 2.0, 2.5, and 3.0 years. (Assume that the 2.5-year swap rate is the average of the 2- and 3-year swap rates.)

Assignment Questions

6.18. The one-year LIBOR rate is 10%. A bank trades swaps where a fixed rate of interest is exchanged for 12-month LIBOR with payments being exchanged annually. Two- and three-year swap rates (expressed with annual compounding) are 11% and 12% per annum. Estimate the two- and three-year LIBOR zero rates.

6.19. Company A, a British manufacturer, wishes to borrow U.S. dollars at a fixed rate of interest. Company B, a U.S. multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following rates per annum (adjusted for differential tax effects):

	Sterling	U.S. dollars
Company A	11.0%	7.0%
Company B	10.6%	6.2%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 15 basis points per annum for each of the two companies.

6.20. Under the terms of an interest rate swap, a financial institution has agreed to pay 10% per annum and to receive three-month LIBOR in return on a notional principal of $100 million with payments being exchanged every three months. The swap has a remaining life of 14 months. The average of the bid and offer fixed rates currently being swapped for three-month LIBOR is 12% per annum for all maturities. The three-month LIBOR rate one month ago was 11.8% per annum. All rates are compounded quarterly. What is the value of the swap?

6.21. Suppose that the term structure of interest rates is flat in the United States and Australia. The USD interest rate is 7% per annum and the AUD rate is 9% per annum. The current value of the AUD is 0.62 USD. Under the terms of a swap agreement, a financial institution pays 8% per annum in AUD and receives 4% per annum in USD. The principals in the two currencies are \$12 million USD and 20 million AUD. Payments are exchanged every year, with one exchange having just taken place. The swap will last two more years. What is the value of the swap to the financial institution? Assume all interest rates are continuously compounded.

6.22. Company X is based in the United Kingdom and would like to borrow \$50 million at a fixed rate of interest for five years in U.S. funds. Because the company is not well known in the United States, this has proved to be impossible. However, the company has been quoted 12% per annum on fixed-rate five-year sterling funds. Company Y is based in the United States and would like to borrow the equivalent of \$50 million in sterling funds for five years at a fixed rate of interest. It has been unable to get a quote but has been offered U.S. dollar funds at 10.5% per annum. Five-year government bonds currently yield 9.5% per annum in the United States and 10.5% in the United Kingdom. Suggest an appropriate currency swap that will net the financial intermediary 0.5% per annum.